Null-cone integral formulation of QFT

Julian Brown, email: gernos@zoho.com

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Abstract

In these preliminary notes we show that there exist null cone integral analogues of both the Dirac equation and the U(1) gauge field. We then explore a generalization of this idea through the introduction of a universal scalar, analogous to the lagrangian density of the Standard Model, from which all known particle equations of motion and interactions can be derived in principle, without recourse to either field derivatives or gauge degrees of freedom. The formulation suggests that at least some of the constants appearing in the Standard Model are related to cosmological quantities such as the total number and mass of particles on the past null cone, and that these are the origin of broken gauge symmetry.

We will employ the conventions used by Feynman *viz.*

$$k^2 \equiv k^\nu k_\nu, \not{A} = \gamma^\mu A_\mu, \not{x} = \gamma^\mu x_\mu, \mathbf{I} = \gamma^0\gamma^0$$

We will denote an infinitesimal element of the past ($t = -r$) null cone Λ by

$$d\Lambda \equiv \delta(t^2 - r^2)d^4x = \frac{d^3\mathbf{r}}{t}$$

It can be shown that, for all $k_0 \neq 0$

$$\int_{-\infty}^{0} e^{-ik_\nu x^\nu} d\Lambda = \frac{1}{k^2} \tag{1}$$

and

$$\int_{-\infty}^{0} ix_\mu e^{-ik_\nu x^\nu} d\Lambda = \frac{2k_\mu}{k^4} \tag{2}$$

(2) and (3) taken together imply that solutions $\psi(x)$ of the Dirac equation for an electron in the presence of an electromagnetic field also satisfy:

$$\int_{-\infty}^{0} [2\mathbf{I} - i\not{x}(m + e\not{A})]\psi(x)d\Lambda = 0 \tag{3}$$

The source equation for a vector gauge field

$$A_\nu = e\int_{-\infty}^{0} \bar{\psi}\gamma_\nu\psi d\Lambda \tag{4}$$

has a tensor analogue

$$\Phi_{\mu\nu} = e\int_{-\infty}^{0} x_\mu \bar{\psi}\gamma_\nu\psi d\Lambda \quad (5)$$

where $\partial^\mu\Phi_{\mu\nu} = gA_\nu$ by virtue of (2). Φ_μ^μ is the conserved charge and the Dirac equation becomes

$$\int_{-\infty}^{0} \gamma_\nu(2\delta^{\mu\nu} + \Phi^{\mu\nu})(m^{-1}\gamma_\mu - ix_\mu)\psi d\Lambda = 0 \quad (6)$$

Consider the following scalar constructed from lorentz and gauge group invariant products of fermionic and gauge field tensors:

$$S = \Phi_{\mu\nu}\Phi^{\mu\nu} + e\int_{-\infty}^{0} \bar{\psi}\gamma_\nu(2\delta^{\mu\nu} + \Phi^{\mu\nu})(m^{-1}\gamma_\mu - ix_\mu)\psi d\Lambda$$

We now posit invariance of S with respect to variations in each of its component fields. Since S contains no derivatives, the equations are considerably simpler than the analogous Euler-Lagrange constraints on the conventional lagrangian density. For example,

$$\frac{\partial S}{\partial\bar{\psi}} = 0$$

yields the Dirac equation (6) and

$$\frac{\partial S}{\partial\Phi_{\mu\nu}} = 0$$

yields (5) plus a small term proportional to m^{-1}:

$$\Phi_{\mu\nu} = e\int_{-\infty}^{0} x_\mu\bar{\psi}\gamma_\nu\psi d\Lambda + em^{-1}\int_{-\infty}^{0} \bar{\psi}\gamma_\mu\gamma_\nu\psi d\Lambda \quad (7)$$

The generalization to non-Abelian groups, fermion doublets and quark triplets is obvious and relatively trivial. Boson mass terms will be of the form:

$$S_M = -M^2\int \Phi_{\mu\nu}\Phi^{\mu\nu} d\Lambda$$

The realtionship of the Higgs field to the m^{-1} term and symmetry breaking will be dealt with in a subsequent paper.