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Diffusion Limited Aggregation and the Spiderweb Distribution of Dark Matter on Galactic Scales

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Abstract

It was recently found that multivariable flows from the ultraviolet (UV) to the infrared (IR) sector of field theory display universal properties. Under the assumption that these flows develop outside equilibrium, they can reproduce the dynamic content of the Standard Model (SM) and shed light on the physics of Dark Matter structures. Here we explore a scenario where the spiderweb patterns of Dark Matter on galactic scales arise from a process resembling three-dimensional Diffusion Limited Aggregation (DLA).

Key words: Strange Attractors, Standard Model, Dark Matter, Cantor Dust, Diffusion Limited Aggregation.

1. Introduction

Pattern formation is a topic of great interest in non-equilibrium statistical physics, complex dynamics and related fields [1-3]. In particular, *diffusion-limited aggregation* (DLA) describes a process where randomly walking particles wander incessantly and eventually stick to dendritic-like clusters in two- or three-dimensional space. In essence, DLA describes a diffusion-driven mechanism of pattern formation endowed with a *multifractal* structure [4-6].

We have recently shown that generic flows connecting the ultraviolet (UV) to the infrared (IR) sectors of field theory and evolving in far-from-equilibrium conditions exhibit universal properties. Specifically, we found that steady-state perturbations near the IR attractor favor formation of Dark Matter (DM) structures while oscillatory perturbations uncover the dynamic composition of the Standard Model [7]. Further pursuing this line of reasoning, this brief report argues that the spiderweb patterns of DM on galactic scales may arise from a process that replicates three-dimensional DLA.

The derivation outlined here is intentionally streamlined for transparency. Building on the framework of ideas detailed in [7-9], we seek to open an unforeseen path to the physics of DM outside the traditional "particle" paradigm. The paper integrates the continuum mean field theory of DLA put forward in [10] with the superfluid picture of DM advanced in [8, 9].

2. Aggregation properties of superfluid phonons

Modeling DM as superfluid phase consisting of axion-like particles offers a number of appealing features [8]. In this proposal, DM "particles" undergo Bose-Einstein condensation and give rise to the superfluid phase inside the galactic cores. The superfluid collective excitations behave as phonons and their coherence properties induce long-range forces. In turn, these forces are able to mimic the predictions of Modified Newtonian Dynamics (MOND) on galactic scales.

It has been long known that superfluidity can be analyzed via the Ginzburg-Landau theory. Specifically, in the weak-coupling approximation, the typical superfluid model consists of a self-interacting complex-scalar field endowed with global $U(1)$ symmetry. In [9], this model was built from the dimensional parameter $\varepsilon = 4 - D << 1$ and led to a superfluid picture of DM bypassing the axion paradigm and referred to as *Cantor Dust*. An alternative strategy was developed in [7] where superfluid phonons are instead described by the real Ginzburg-Landau equation

$$z'_{\tau} = \varepsilon z - uz|z|^2 + \eta \frac{\partial^2 z}{\partial x^2} \tag{1}$$

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In planar geometry, ρ is the "cluster" density of phonons defined on a square cell area of Euclidean length a

$$\rho(\varepsilon) = \frac{1}{a^{2-\varepsilon}} \tag{2}$$

The phonon number per square cell is given by

$$N = \frac{1}{a^{2-\varepsilon}} \int_{\Sigma} \rho(s) d^{1-\varepsilon} s \tag{3}$$

where s is the elemental area and Σ the integration domain.

The key observation of this work is to identify the dimensional control parameter $\varepsilon = 2 - D$ by the cluster density of phonons (2). If μ is the probability density describing the random walk of phonons due to the diffusion term $\Delta_{\varepsilon}\mu$ and axial symmetry of the aggregation process is assumed,

$$\frac{\partial \mu}{\partial \tau} = -\rho\mu + \frac{a^2}{4} \Delta_{\varepsilon} \mu (1 - \rho) \tag{4}$$

in which the standard Laplacian is replaced by its fractional counterpart

$$\Delta_{\varepsilon} = \frac{\partial^{2-\varepsilon}}{\partial r^{2-\varepsilon}} + \frac{1}{r^{1-\varepsilon}} \frac{\partial^{1-\varepsilon}}{\partial r^{1-\varepsilon}} \qquad (5)$$

To streamline the presentation and drive home the essential point of the argument, we assume in what follows that the (5) is reasonably well approximated by its standard formulation on smooth spacetime, that is, $\Delta_{\varepsilon} \to \Delta$.

Term by term identification of (4) with (1) leads to ($\mu \to z,\ \rho \to -\varepsilon$):

$$z'_{\tau} = -\rho z + \eta \frac{\partial^2 z}{\partial x^2} \qquad (6)$$

The time-averaged space distribution of phonons can be presented as [10]

$$Z(r) = \frac{1}{\tau^0} \int_0^{\tau_0} z(r,\tau)\, d\tau \qquad (7)$$

where τ^0 is the phonon lifetime. Setting the boundary conditions as

$$z(\tau^0) = 0 \qquad (8)$$

$$\frac{\partial}{\partial r} Z(r)\Big|_{r=0,\, R} = 0 \qquad (9)$$

and passing to a discrete time representation leads to the following system of equations

$$\varepsilon_{n+1} = \varepsilon_n + C\, Z(\varepsilon + \Delta\varepsilon) \qquad (10)$$

$$-z(r,0) = \Delta Z(r) - \varepsilon_n(r)[Z(r) + \frac{a^2}{4} \Delta Z(r)] \qquad (11)$$

where

$$C = r\frac{a}{\int Z(\varepsilon + \Delta\varepsilon) r dr} \tag{12}$$

Here, n is the iteration step index at which the cluster becomes covered with a new layer of adsorbed random walkers.

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3. References

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