A SIMPLE, DIRECT PROOF OF FERMAT'S LAST THEOREM

PHIL AARON BLOOM; BRAINEMAIL1@GMAIL.COM; V.3

ABSTRACT. An open problem is proving FLT simply for each and every value of $n \in \mathbb{N}, n > 2$. Our *direct proof* (not by way of contradiction) of FLT is based on our algebraic identity, denoted, for convenience, as $r^n + s^n = t^n$ with $r, s, t > 0$ as functions of variables. We infer that $\{(r,s,t)|r,s,t \in \mathbb{N}, r^n + s^n + t^n\} = \{(x,y,z)|x,y,z \in \mathbb{N}, x^n + y^n = z^n\}$ for $n \in \mathbb{N}, n > 2$. In addition, we show, for integral values of $n > 2$, that $\{(r,s,t)|r,s,t \in \mathbb{N}, r^n + s^n = t^n\} = \varnothing$. Hence, for values of $n \in \mathbb{N}, n > 2$, it is true that $\{(x,y,z)|x,y,z \in \mathbb{N}, x^n + y^n = z^n\} = \varnothing$.

1. Introduction

FLT states, for $n \in \mathbb{N}, n > 2$, $x, y, z \in \mathbb{N}, x, y, z > 0$ that $x^n + y^n = z^n$ *does not hold*. It is well known that a *simple* proof of FLT for *every* $n \in \mathbb{N}, n > 2$ is lacking.

For $n \in \mathbb{N}$: We use *basics* to devise a *direct proof*, not the *expected* BWOC.

An *identity* with no rational triples for any given $n \in \mathbb{N}, n > 2, q \in \mathbb{Q}$ since term $(4q^n)^{\frac{1}{n}}\alpha$ of this identity reduces to $2^{\frac{2}{n}}q\alpha$, a term that for $n > 2$ is irrational, is :

$$\left((4q^n)^{\frac{1}{n}}\,\alpha\right)^n + \left((p - 2q^n)^{\frac{1}{n}}\,\alpha\right)^n = \left((p + 2q^n)^{\frac{1}{n}}\,\alpha\right)^n. \tag{1}$$

Conditions : For $n \in \mathbb{N}, p \in \mathbb{R}, \alpha, q \in \mathbb{Q}, \alpha, n, p, q > 0$ such that $p > 2q^n$.

Denote throughout this paper, for convenience : r for $(4q^n)^{\frac{1}{n}}\alpha$; s for $(p-2q^n)^{\frac{1}{n}}\alpha$; t for $(p+2q^n)^{\frac{1}{n}}\alpha$, with (r,s,t) functions of p, q, such that, for $n > 0, r, s, t \in \mathbb{Q}$, with $r, s, t > 0$ for which $r^n + s^n = t^n$ holds. Note : We do not intend that arbitrary rational values of p, q in (1) would yield rational (r,s,t). We intend, for $n > 2, q \in \mathbb{Q}$, that $\{r|r,s,t \in \mathbb{Q}, r^n+s^n = t^n\} = \varnothing$ would imply $\{x|x,y,z \in \mathbb{Q}, x^n+y^n = z^n\} = \varnothing$.

Per Sect. 2, below, $q \in \mathbb{Q}$, thus, for $n > 2$: $\{(r,s,t)|r,s,t \in \mathbb{Q}, r^n+s^n = t^n\} = \varnothing$.

Some *alternate identity*, not as simple as (1), can also work in our argument with *alternate* r, s, t, but, for $n = 1, 2$, the following *remains true* with $r, s, t, x, y, z > 0$: $\{(r,s,t)|r,s,t \in \mathbb{N}, r^n+s^n = t^n\} = \{(x,y,z)|x,y,z \in \mathbb{N}, x^n+y^n = z^n\}$; equation (1) is true for $n = 1, 2$, but *solely* with $q \in \mathbb{Q}$, $q = \frac{r\alpha}{4}, \frac{r\alpha}{2}$, respectively; consequently, $\{(r,s,t)|r,s,t \in \mathbb{N}, r^n+s^n = t^n\} = \{x,y,z|x,y,z \in \mathbb{N}, x^n+y^n = z^n\}$ would be false should, instead, $q \in \mathbb{R} - \mathbb{Q}$. Therefore, we must exclude $q \in \mathbb{R} - \mathbb{Q}$ from our proof.

That $\{(r,s,t)|r,s,t \in \mathbb{N}, r^n + s^n = t^n\} = \{x,y,z|x,y,z \in \mathbb{N}, x^n + y^n = z^n\}$ is a true statement is a fact that we establish in section 3, below, for $n \in \mathbb{N}, n > 2$, with unrestricted p, with q restricted to rational values, and with $r, s, t, x, y, t > 0$.

So, for $n > 2$: Equation $\{(x,y,z)|x,y,z \in \mathbb{N}, x^n+y^n = z^n\} = \varnothing$ (*which is FLT*) is true since, per above, it is true that $\{(r,s,t)|r,s,t \in \mathbb{N} \subset Q, r^n + s^n = t^n\} = \varnothing$.

Date: December 11, 2018.

2. Our Very Simple, Very Brief, Direct Proof of FLT

For $n > 2$, we want to infer the conditions for $\{(x, y, z)|x, y, z \in \mathbb{N}, x^n + y^n = z^n\}$.

Consider *for any given* $n \in \mathbb{N}, n > 0$, two triples, with $p \in \mathbb{R}, q \in \mathbb{Q}$ for (r, s, t) :
Triple (r, s, t) such that $r, s, t \in \mathbb{Q}, r, s, t > 0$, for which $r^n + s^n = t^n$ holds, and triple (x, y, z) such that $x, y, z \in \mathbb{Q}, x, y, z > 0$ for which $x^n + t^n = z^n$ holds.

Arbitrarily choose a positive rational value of $(p + 2q^n)^{\frac{1}{n}}$, or t/α, *denoted* as $j \in \mathbb{Q}$, for which $(r/\alpha)^n + (s/\alpha)^n = (t/\alpha)^n$ holds, for which, with equal $q \in \mathbb{Q}, p \in \mathbb{R}$, there is a corresponding rational value of $(p - 2q^n)^{\frac{1}{n}}$ or s/α, *denoted* as $k \in \mathbb{Q}$.
For $n = 1, 2$, it is easy to choose $(j, k) \in \mathbb{Q}$ with equal $q \in \mathbb{Q}$.
For $n > 2$, this choice is difficult but seems possible at this point since solving simultaneously $(p + 2q^n)^{\frac{1}{n}} = j$ with $(p - 2q^n)^{\frac{1}{n}} = k$ yields $q \in \mathbb{Q} = \left(\frac{j^n + k^n}{4}\right)^{\frac{1}{n}}$.

With such $(p + 2q^n)^{\frac{1}{n}} \in \mathbb{Q}$ or $t/\alpha \in \mathbb{Q}$: Since α has all positive rational values, α varies such that such $(t/\alpha)(\alpha \in \mathbb{Q})$ or $t \in \mathbb{Q}$ takes every value of such $z \in \mathbb{Q}$.
Clearly, by definition, such $z \in \mathbb{Q}$ takes every value of such $t \in \mathbb{Q}$.
Thus, $\{t|(r, s, t), r, s, t \in \mathbb{Q}, r^n + s^n = t^n\} = \{z|(x, y, z), x, y, z \in \mathbb{Q}, x^n + y^n = z^n\}$ is true, *with restriction to* $q \in \mathbb{Q}$, with either both sets empty or both sets nonempty.

With $(p - 2q^n)^{\frac{1}{n}} \in \mathbb{Q}$ or $s/\alpha \in \mathbb{Q}$, corresponding to $(p + 2q^n)^{\frac{1}{n}} \in \mathbb{Q}$: With fixed q, unrestricted real $p \in \mathbb{R}$ varies such that such $s/\alpha \in \mathbb{Q}$ takes every value of such $y \in \mathbb{Q}$. Such $(s/\alpha \in \mathbb{Q})(\alpha)$ yields $s \in \mathbb{Q}$, which still takes every value of such $y \in \mathbb{Q}$.
By definition, clearly, such $y \in \mathbb{Q}$ takes every value of such $s \in \mathbb{Q}$.
Thus, $\{s|(r, s, t), r, s, t \in \mathbb{Q}, r^n + s^n = t^n\} = \{y|(x, y, z), x, y, z \in \mathbb{Q}, x^n + y^n = z^n\}$ is true with either both sets empty or both sets nonempty.

Due to identical forms, $r^n + s^n = t^n$; $x^n + y^n = z^n$: Such $r \in \mathbb{Q}$ or $(4q^n)^{\frac{1}{n}}\alpha \in \mathbb{Q}$, is *constrained* to have values for which the following equation of sets holds :
$\{r|(r, s, t), r, s, t \in \mathbb{Q}, r^n + s^n = t^n\} = \{x|(x, y, z), x, y, z \in \mathbb{Q}, x^n + y^n = z^n\}$,
for which either both sets are empty or both sets are nonempty.

Thus, for $n > 0$, it is true, with $q \in \mathbb{Q}$ and both sets empty or nonempty, that :
$\{(r, s, t)|r, s, t \in \mathbb{Q}, r^n + s^n = t^n\} = \{(x, y, z)|x, y, z \in \mathbb{Q}, x^n + y^n = z^n\}$.

3. Results and Conclusion

For $n \in \mathbb{N}, n > 2$, per Sect. 1 : $\{(r, s, t)|r, s, t \in \mathbb{N} \subset Q, r^n + s^n = t^n\} = \varnothing$.

For $n \in \mathbb{N}, n > 2$, per Sect. 2 :
$\{(r, s, t)|r, s, t \in \mathbb{N} \subset Q, r^n + s^n = t^n\} = \{(x, y, z)|x, y, z \in \mathbb{N} \subset Q, x^n + y^n = z^n\}$.
Hence, for any given $n \in \mathbb{N}, n > 2$: $\{(x, y, z)|x, y, z \in \mathbb{N}, x^n + y^n = z^n\} = \varnothing$.

In other words, for any given $n \in \mathbb{N}, n > 2$, the following is a true statement :
Equation $x^n + y^n = z^n$ does not hold for (x, y, z) with $x, y, z \in \mathbb{N}, x, y, z > 0$, which is a common definition of Fermat's last theorem (FLT).

QED.