Refutation of definable modal operators on stable set lattices

Abstract: We evaluate definable modal operators on stable set lattices. None of the definitions or implementations is tautologous, hence refuting the operators.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal. The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨; - Not Or; & And, ∧; \ Not And;
> Imply, greater than, →; < Not Imply, less than, ∈;
= Equivalent, ≡; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊; # necessity, for every or all, ∀, □;
~(y < x) (x ≤ y), (x ⊆y).

From: Goldblatt, R. (2018). Definable operators on stable set lattices.
arxiv.org/pdf/1812.01264.pdf rob.goldblatt@msor.vuw.ac.nz

> The key idea is that of a first-order definable operation on a stable set lattice, an idea that goes to the heart of Kripke's semantical interpretation of the modalities □ and ◊. On the algebra of subsets of a Kripke frame (X, R), the modal connectives can be interpreted as operations assigning to each set A ⊆X the sets

[LET p, q, r, s: A, R, x, y.]

□A = {x : ∀y(xRy → y ∈A)} (2.1.1)

#p=(r>((r&(q&#s))>(r<p))) ; **F**N**F**N **F**N**F**N **F**N**F**N **F**N**FF** (2.1.2)

and ◊A = {x : ∃y(xRy & y ∈A)}. (2.2.1)

%p=(r>((r&(q&%s))>(r<p))) ;
CTCT CTCN CTCT CTC**F** (2.2.2)

Example 3.2. Modal operators.

[LET p, r, t, x, y, z: A, R, T, x, y, z.]

□A = {x : ∀y[∀z(z ∈A → zRy) → xTy]} (3.2.1.1)

#p=(x>(((#z<p)>(#z&(r&#y)))>(x&(t&#y)))) ;
TCTC TCTC TCTC TCTC, CCCC CCCC CCCC CCCC,
FN**F**N **F**N**F**N **F**N**F**N **F**N**F**N, CCCC TCTC CCCC TCTC,
CTCT CTCT CTCT CTCT (3.2.1.2)

$$\Diamond A = \{x : \forall y[\ \forall z(z \in A \rightarrow zTy) \rightarrow xRy\ ]\}, \tag{3.2.2.1}$$

```
%p=(x>(((#z<p)>(#z&(t&#y)))>(x&(r&#y)))) ;
      CTCT CTCT CTCT CTCT, FFFF FNFN FFFF FNFN,
      NFNF FNFN NFNF FNFN                                  (3.2.2.2)
```

Eqs. 2 and 3 as rendered are *not* tautologous, hence refuting definable modal operators on stable set lattices.