

Refutation of the constructive Craig interpolation theorem by Feferman

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Abstract: We evaluate the Craig constructive interpolation theorem, find a mistake in a Craig lemma as rendered by Feferman, and refute the theorem as not constructive.

We assume the method and apparatus of Meth8/VL4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET \sim Not, \neg ; + Or, \vee ; - Not Or; & And, \wedge ; \ Not And;
 $>$ Imply, greater than, \rightarrow ; $<$ Not Imply, less than, \Leftarrow
 $=$ Equivalent, \equiv ; @ Not Equivalent, \neq ;
 $\%$ possibility, for one or some, \exists, \diamond ; # necessity, for every or all, \forall, \square ;
 $\sim(y < x)$ ($x \leq y$), ($x \subseteq y$).

From: en.wikipedia.org/wiki/Craig_interpolation;

Remark 0: We use only four variables to minimize table results to 4-rows or 16-values (instead of 512-rows or 2048-values) . Hence we avoid direct assignment of ϕ, ψ as separate variables.

LET $p, q, r, s:$ P, Q, R, T.

In propositional logic, let

$$\phi = \sim(P \wedge Q) \rightarrow (\sim R \wedge Q) \tag{1.1.1}$$

$$\sim(p \& q) > (\sim r \& q); \quad \mathbf{FFTT \ FFFT \ FFTT \ FFFT} \tag{1.1.2}$$

$$\psi = (T \rightarrow P) \vee (T \rightarrow \sim R). \tag{1.2.1}$$

$$(s > p) + (s > \sim r); \quad \mathbf{TTTT \ TTTT \ TTTT \ FTFT} \tag{1.2.2}$$

Then ϕ tautologically implies ψ , but only because both are not tautologous. (1.3.1)

$$(\sim(p \& q) > (\sim r \& q)) > ((s > p) + (s > \sim r)); \quad \mathbf{TTTT \ TTTT \ TTTT \ TTTT} \tag{1.3.2}$$

Eq. 1.3.2 refutes the Craig interpolation theorem because both Eqs.1.1.2 and 1.2.2 are *not* tautologous.

Remark 1: Eq. 1.3 is a tautology via contradiction implying contradiction (**F** $>$ **F**=**T**). This form of proof is not constructive in an affirmative or positive sense. A much longer constructive proof exists, but it can be minimized by its use of *induction*.

From: Feferman, S. (2008). Harmonious Logic: Craig's Interpolation Theorem and its Descendants
math.stanford.edu/~feferman/papers/Harmonious%20Logic.pdf

[Here \vdash is validity in classical first order logic with equality (FOL), ϕ, ψ, θ are sentences, and $R, S,$ and T are sequences of relation symbols for which the sequence S is non-empty.]

A common statement of Craig's theorem (initially referred to by him as a lemma) goes as follows:

[LET $r, s, t, w, x, y: R, S, T, \phi, \psi, \theta.$]

Suppose $\vdash \phi(R, S) \rightarrow \psi(S, T)$. Then there is a $\theta(S)$ such that $\vdash \phi(R, S) \rightarrow \theta(S)$ and $\vdash \theta(S) \rightarrow \psi(S, T)$. (2.1)

Remark 1.1: Again, the antecedent and consequent are both not tautologous, hence the product of a tautology, as in Remark 1 is not constructive.

$((w \& (r \& s)) \> (x \& (s \& t))) = (p = p) \>$
 $((y \& s) \> (((w \& (r \& s)) \> (y \& s)) = (p = p)) \& (((y \& s) \> (x \& (s \& t))) = (p = p)))) ;$
TTTT TTTT TTTT TTTT (2.2)

Accounting for the unclear writing of Craig (and Feferman), the lemma of Eq. 2.2 as rendered refutes the Craig interpolation as a constructive theorem.