

## Refutation of AGM postulates and Levi and Harper bridging principles

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**Abstract:** We evaluate the AGM logic system in eight postulates and two bridging principles. The postulates named success, inclusion, vacuity, and inconsistency and the Levi and Harper bridging principles are *not* tautologous, hence refuting the AGM logic system.

We assume the method and apparatus of Meth8/VL4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal. The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET: ~ Not, ¬; + Or, ∨; - Not Or; & And, ∧, ∩; \ Not And;  
 > Imply, greater than, →; < Not Imply, less than, ∈  
 = Equivalent, ≡; @ Not Equivalent, ≠;  
 % possibility, for one or some, ∃, ∅; # necessity, for every or all, ∀, □;  
 ~(y < x) (x ≤ y), (x ⊆ y); (p=p) Tautology.

From: en.wikipedia.org/wiki/Belief\_revision; philarchive.org/archive/LINEDD cite

The AGM postulates (named after the names of their proponents, Alchourrón, Gärdenfors, and Makinson) are properties that an operator that performs revision should satisfy in order for that operator to be considered rational. The considered setting is that of revision, that is, different pieces of information referring to the same situation. Three operations are considered: expansion (addition of a belief without a consistency check), revision (addition of a belief while maintaining consistency), and contraction (removal of a belief).

The first six postulates are called "the basic AGM postulates". In the settings considered by Alchourrón, Gärdenfors, and Makinson, the current set of beliefs is represented by a deductively closed set of logical formulae K called belief base, the new piece of information is a logical formula P, and revision is performed by a binary operator \* that takes as its operands the current beliefs and the new information and produces as a result a belief base representing the result of the revision. The + operator denoted expansion: K+P is the deductive closure of K∪{P}. The AGM postulates for revision are:

LET: p, q, r, s: P, Q, K, consistent

Closure: K\*P is a belief base (i.e., a deductively closed set of formulae) (1.1)

(r&p)>(r+p); TTTT TTTT TTTT TTTT (1.2)

Success: P∈K\*P (2.1)

p<(r&p); FTFT FFFF FTFT FFFF (2.2)

$$\text{Inclusion: } K * P \subseteq K + P \quad (3.1)$$

$$\sim((r+p) < (r \& p)) = (p = p) ; \quad \mathbf{TFTF \ FTFT \ TFTF \ FTFT} \quad (3.2)$$

$$\text{Vacuity: If } (\neg P) \notin K, \text{ then } K * P = K + P \quad (4.1)$$

$$\sim(\sim p < r) > ((r \& p) = (r + p)) ; \quad \mathbf{TFTF \ FTFT \ TFTF \ FTFT} \quad (4.2)$$

$$K * P \text{ is inconsistent only if } P \text{ is inconsistent or } K \text{ is inconsistent} \quad (5.1)$$

$$((p > \sim s) + (q > \sim s)) > ((r \& p) > \sim s) ; \quad \mathbf{TTTT \ TTTT \ TTTT \ TFTT} \quad (5.2)$$

$$\text{Extensionality: If } P \text{ and } Q \text{ are logically equivalent, then } K * P = K * Q \quad (6.1)$$

$$(p = q) > ((r \& p) = (r \& q)) ; \quad \mathbf{TTTT \ TTTT \ TTTT \ TTTT} \quad (6.2)$$

$$K * (P \wedge Q) \subseteq (K * P) + Q \quad (7.1)$$

$$\sim(((r \& p) + q) < (r \& (p \& q))) = (p = p) ; \quad \mathbf{TTFF \ TFFT \ TTFF \ TFFT} \quad (7.2)$$

$$\text{If } (\neg Q) \notin K * P \text{ then } (K * P) + Q \subseteq K * (P \wedge Q) \quad (8.1)$$

$$(\sim q < (r \& p)) > \sim((r \& (p \& q)) < ((r \& p) + q)) ; \quad \mathbf{TTTT \ TTTT \ TTTT \ TTTT} \quad (8.2)$$

From: [philarchive.org/archive/LINEDD](http://philarchive.org/archive/LINEDD)

AGM also contains ... the following bridging principles:

$$\text{LET: } p, q: G, \alpha$$

$$\text{Levi identity: } (G * \alpha) = (G - \neg \alpha) + \alpha \quad (10.1)$$

$$(p \& q) = ((p \sim \sim q) + q) ; \quad \mathbf{TFTT \ TTFT \ TTFT \ TTFT} \quad (10.2)$$

The Levi identity says that the result of revising the belief set  $G$  by the sentence  $\alpha$  equals the result of first making room for  $\alpha$  by (if necessary) contracting  $G$  with  $\neg \alpha$  and then expanding the result with  $\alpha$ .

$$\text{Harper identity: } (G - \alpha) = (G * \alpha) \cap (G * \neg \alpha) \quad (11.1)$$

$$(p - q) = ((p \& q) \& (p \& \sim q)) ; \quad \mathbf{FTTT \ FTTT \ FTTT \ FTTT} \quad (11.2)$$

The Harper identity says that the result of contracting  $\alpha$  from  $G$  is the common part of  $G$  revised with  $\alpha$  and  $G$  revised with  $\neg \alpha$ .

Eqs. for postulates named success, inclusion, vacuity, and inconsistency and the Levi and Harper bridging principles are *not* tautologous, hence refuting the AGM logic system.