Refutation of Lyndon interpolation

Abstract: We evaluate the Lyndon interpolation on the logic **GL**. Each is *not* tautologous, and the combination is *not* tautologous, hence rendering both refuted.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal. The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET: ~ Not, ¬ ; + Or, ∨, ∪; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →; < Not Imply, less than, ∈;
= Equivalent, ≡; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊; # necessity, for every or all, ∀, □;
~(y < x) (x ≤ y), (x ⊆y); (p=p) Tautology.

From: Kurahashi, T. (2018). Uniform Lyndon interpolation property in propositional modal logics. arxiv.org/pdf/1809.00943.pdf kurahashi@n.kisarazu.ac.jp

LET p, q, r, s: ϕ phi, ψ psi, v, theta θ.

Definition 2.2. The least normal logic is called **K**.

K = □(p → q) → (□p → □q) (2.2.0.1)

#(p>q)>(#p>#q) ; TTTT TTTT TTTT TTTT (2.2.0.2)

GL = **K** + {□(□p → p) → □p} (2.2.1.1)

(#(p>q)>(#p>#q))&(#(#p>p)>#p) ;
CTCT CTCT CTCT CTCT (2.2.1.2)

Eq. 2.2.1.2 as **GL** is *not* tautologous. This means logic **GL** is *not* a logic proved as a theorem.

Definition 2.5. We say a logic L enjoys the Lyndon interpolation property (LIP) if for any formulas ϕ and ψ, if L ⊢

ϕ → ψ, (2.5.0.1)

p>q ; T**F**TT T**F**TT TFTT TFTT (2.5.0.2)

then there exists a formula θ satisfying the following properties:

1. v+(θ) ⊆v+(ϕ) ∩ v+(ψ); (2.5.1.1)

~(((r&p)&(r&q))<(r&s)) = (p=p) ; TTTT TTT**F** TTTT TTTT (2.5.1.2)

2. $v-(\theta) \subseteq v-(\phi) \cap v-(\psi)$; (2.5.2.1)

~(((~r&p)&(~r&q))<(~r&s)) = (p=p) ;
TTT**F** TTTT TTTT TTTT (2.5.2.2)

3. $L \vdash \phi \rightarrow \theta$; (2.5.3.1)

p>s ; T**F**T**F** T**F**T**F** TTTT TTTT (2.5.3.2)

4. $L \vdash \theta \rightarrow \psi$. (2.5.4.1)

s>q ; TTTT TTTT **FF**TT **FF**TT (2.5.4.2)

Such a formula θ is said to be a Lyndon interpolant of $\phi \rightarrow \psi$ in L.

The argument becomes: $\phi \rightarrow \psi$ implies that if $(v+(\theta) \subseteq v+(\phi) \cap v+(\psi))$ and $(v-(\theta) \subseteq v-(\phi) \cap v-(\psi))$ and $\phi \rightarrow \theta$ and $\theta \rightarrow \psi$, then θ as Lyndon interpolant. (2.5.5.1)

(p>q)>(((~(((~r&p)&(~r&q))<(~r&s))
&~(((r&p)&(r&q))<(r&s)))&((p>s)&(s>q)))>s) ;
FT**F**T **F**T**F**T TTTT TTTT (2.5.5.2)

Eq. 2.5.5.2 as rendered is *not* tautologous. This means the Lyndon interpolation is refuted.

Remark 5: To assert that the non-tautologous Lyndon interpolation applies to the non-tautologous logic **GL** is a further mistake. (5.0.1.1)

((p>q)>(((~(((~r&p)&(~r&q))<(~r&s))&
~(((r&p)&(r&q))<(r&s)))&((p>s)&(s>q)))>s)) &
((#(p>q)>(#p>#q))&(#(#p>p)>#p)) ;
FT**F**T **F**T**F**T CTCT CTCT (5.0.1.2)