Neutrosophic Shortest Path Problem

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Abstract. Neutrosophic set theory provides a new tool to handle the uncertainties in shortest path problem (SPP). This paper introduces the SPP from a source node to a destination node on a neutrosophic graph in which a positive neutrosophic number is assigned to each edge as its edge cost. We define this problem as neutrosophic shortest path problem (NSSPP). A simple algorithm is also introduced to solve the NSSPP. The proposed algorithm finds the neutrosophic shortest path (NSSP) and its corresponding neutrosophic shortest path length (NSSPL) between source node and destination node. Our proposed algorithm is also capable to find crisp shortest path length (CrSPL) of the corresponding neutrosophic shortest path length (NSSPL) which helps the decision maker to choose the shortest path easily. We also compare our proposed algorithm with some existing methods to show efficiency of our proposed algorithm. Finally, some numerical experiments are given to show the effectiveness and robustness of the new model. Numerical and graphical results demonstrate that the novel methods are superior to the existing method.

Keywords: Trapezoidal neutrosophic fuzzy numbers; scoring, accuracy and certainty index, shortest path problem

1 Introduction

Let $G = (V, E)$ be a graph, where V is a set of all the nodes (or vertices) and E is a set of all the edges (or arcs). The aim of the shortest path problem (SPP) is to find a path between two nodes and optimizing the weight of the path. The SPP is known as one of the well-studied fields in the area operations research and mathematical optimization and it is commonly encountered in wide array of practical applications including road network [1], flow shop scheduling [2], routing problems [3], transportation planning [4], geographical information system(GIS) field [5], optimal path[6-7] and so on.

There are several methods for solving traditional SPP such as Dijkstra [8] algorithm or the label-correcting Bellman [9] algorithm. Due to uncertain factors in real-world problems, such as efficiency, expense, and path capacity variation, we must consider SPP with imprecise information. Under some circumstances, an approximate method applies fuzzy numbers to solve SPP, called Fuzzy-SPP (FSPP). Many researchers have focused on FSPP and intuitionistic FSPP (IFSPP) formulations and solution approaches. Dubois and Prade [10] first introduced FSPP. Later, different approaches were presented by various researchers/scientists to evaluate the FSPP. Some of them are as follows; Keshavarz and Khorram [11] used the highest reliability, Deng et al. [12] suggested extended Dijkstra Principle technique, Hassanzadeh et al. [13], and Syarif et al. [14] proposed a genetic algorithm model, Ebrahimnejad et al. [15] using the artificial bee colony model, Li et al.[16]; Zhong and Zhou [17] used neural networks for finding FSPP. Moreover, Motameni and Ebrahimnejad [18] considered constraint SPP, Mukherjee [19], Geetharamani and Jayagowri [20] and Biswas et al. [21] considered the IFSPP. In recent years, research on this subject has increased and that is of continuing interest such as Kristianto et al.[22], Zhang et al.[23], Mukherjee [24], Huang and Wang [25], Dey et al. [26], Niroomand et al. [27], Rashmanlou et al. [28], Mali and Gautam [29], Wang et al. [30], Yen and Cheng [31] and so on.

Recently, neutrosophic set (NS) theory is proposed by Smarandache [32-33], and this is generalised from the fuzzy set [34] and intuitionistic fuzzy set [35]. NS deals with uncertain, indeterminate and incongruous data where the indeterminacy is quantified explicitly. Moreover, falsity, indeterminacy and truth membership are completely independent. It overcomes some limitations of the existing methods in depicting uncertain decision information. Some extensions of NSs, including interval NS [36-38] , bipolar NS[39], single-valued NS [40-44], multi-valued NS [45-47], neutrosophic linguistic set [48-49], rough neutrosophic set [50-62], triangular fuzzy neutrosophic set [63], and neutrosophic trapezoidal set [64-67] have been proposed and applied to solve various problems. However, to the best of our knowledge, there are few methods which deal with NSSPP. Recently, Broumi et al.[68-71] proposed some models to solve SPP in the neutrosophic environment. Broumi et al. [68-71], proposed a new method for the TrNSSPP and TNSSPP. However, the mentioned methods [68-71] have some shortcomings and are not valid. In this paper, for finding NSSPP, the shortcomings of the mentioned models are pointed out, and a new method is proposed for the same.

2 Preliminaries

Definition 2.1: [72]: Let 1 is a special NS on the real number set R, whose truth-MF $\mu_{\tilde{a}}(x)$, indeterminacy-MF $\nu_{\tilde{a}}(x)$, and falsity-MF $\lambda_{\tilde{a}}(x)$ are given as follows:

$$\mu_{\tilde{a}}(x)=\begin{cases}\dfrac{T_{\tilde{a}}\left(x-\tilde{a}_T\right)}{\left(\tilde{a}_I-\tilde{a}_T\right)} & \tilde{a}_T\le x\le \tilde{a}_I,\\ T_{\tilde{a}} & \tilde{a}_I\le x\le \tilde{a}_P,\\ \dfrac{T_{\tilde{a}}\left(\tilde{a}_S-x\right)}{\left(\tilde{a}_S-\tilde{a}_P\right)} & \tilde{a}_P\le x\le \tilde{a}_S,\\ 0 & otherwise.\end{cases} \tag{1}$$

$$\nu_{\tilde{a}}(x)=\begin{cases}\dfrac{\left(\tilde{a}_I-x+I_{\tilde{a}}\left(x-\tilde{a}_T\right)\right)}{\left(\tilde{a}_I-\tilde{a}_T\right)} & \tilde{a}_T\le x\le \tilde{a}_I,\\ I_{\tilde{a}} & \tilde{a}_I\le x\le \tilde{a}_P,\\ \dfrac{\left(x-\tilde{a}_P+I_{\tilde{a}}\left(\tilde{a}_S-x\right)\right)}{\left(\tilde{a}_S-\tilde{a}_P\right)} & \tilde{a}_P\le x\le \tilde{a}_S,\\ 1 & otherwise.\end{cases} \tag{2}$$

$$\lambda_{\tilde{a}}(x)=\begin{cases}\dfrac{\left(\tilde{a}_I-x+F_{\tilde{a}}\left(x-\tilde{a}_T\right)\right)}{\left(\tilde{a}_I-\tilde{a}_T\right)} & \tilde{a}_T\le x\le \tilde{a}_I,\\ F_{\tilde{a}} & \tilde{a}_I\le x\le \tilde{a}_p,\\ \dfrac{\left(x-\tilde{a}_P+F_{\tilde{a}}\left(\tilde{a}_S-x\right)\right)}{\left(\tilde{a}_S-\tilde{a}_P\right)} & \tilde{a}_P\le x\le \tilde{a}_S,\\ 1 & otherwise.\end{cases} \tag{3}$$

The graphical representation of the TrNS number $\tilde{a}=\left\langle\left[\tilde{a}_T,\tilde{a}_I,\tilde{a}_P,\tilde{a}_S\right],\left(T_{\tilde{a}},I_{\tilde{a}},F_{\tilde{a}}\right)\right\rangle$ is shown in Fig. 1, where, the burgundy colour graph show truth-MF, the yellow colour graph shows indeterminacy-MF, and the red colour graph shows the falsity-MF. Blackline represent the truth value, the cyan line represents the indeterminacy value, and the blue line represents the falsity value (here, we consider $T_{\tilde{a}}>I_{\tilde{a}}>F_{\tilde{a}}$).

Additionally, when $\tilde{a}_T>0$, $\tilde{a}=\left\langle\left[\tilde{a}_T,\tilde{a}_I,\tilde{a}_P,\tilde{a}_S\right],\left(T_{\tilde{a}},I_{\tilde{a}},F_{\tilde{a}}\right)\right\rangle$ is called a positive TrNS number. Similarly, when $\tilde{a}_S\le 0$, $\tilde{a}=\left\langle\left[\tilde{a}_T,\tilde{a}_I,\tilde{a}_P,\tilde{a}_S\right],\left(T_{\tilde{a}},I_{\tilde{a}},F_{\tilde{a}}\right)\right\rangle$ becomes a negative TrNS number. When $0\le\tilde{a}_T\le\tilde{a}_I\le\tilde{a}_P\le\tilde{a}_S\le 1$ and $T_{\tilde{a}},I_{\tilde{a}},F_{\tilde{a}}\in[0,1]$, $\tilde{a}$ is called a normalised TrNS number. When $I_{\tilde{a}}=1-T_{\tilde{a}}-F_{\tilde{a}}$, the TrNS number is reduced to triangular intuitionistic fuzzy numbers (TrIFN). When $\tilde{a}_I=\tilde{a}_P,\tilde{a}=\left\langle\left[\tilde{a}_T,\tilde{a}_I,\tilde{a}_P,\tilde{a}_S\right],\left(T_{\tilde{a}},I_{\tilde{a}},F_{\tilde{a}}\right)\right\rangle$ transforming into a TNS number. When $I_{\tilde{a}}=0,F_{\tilde{a}}=0$, a TrNS number is reduced to generalised TrFN, $\tilde{a}=\left\langle\left[\tilde{a}_T,\tilde{a}_I,\tilde{a}_P,\tilde{a}_S\right],T_{\tilde{a}}\right\rangle$.

Definition 2.2: [40] : Let X be a space point or objects, with a genetic element in X denoted by x. A single-valued NS, V in X is characterised by three independent parts, namely truth-MF T_V, indeterminacy-MF I_V and falsity-MF F_V, such that $T_V:X\to[0,1], I_V:X\to[0,1],$ *and* $F_V:X\to[0,1]$.
Now, V is denoted as $V=\left\{<x,\left(T_V(x),I_V(x),F_V(x)\right)> \;|\, x\in X\right\}$, satisfying $0\le T_V(x)+I_V(x)+F_V(x)\le 3$.

Definition 2.3: [72]: Let $\hat{r}^N=\left\langle\left[\hat{r}_T,\hat{r}_I,\hat{r}_M,\hat{r}_E\right],\left(T_{\hat{r}},I_{\hat{r}},F_{\hat{r}}\right)\right\rangle$ and $\hat{s}^N=\left\langle\left[\hat{s}_T,\hat{s}_I,\hat{s}_M,\hat{s}_E\right],\left(T_{\hat{s}},I_{\hat{s}},F_{\hat{s}}\right)\right\rangle$ be two arbitrary TrNSNs, and $\theta\ge 0$; then arithmetic operation on TrNS are as follows:

$$\hat{r}^N \oplus \hat{s}^N = \left\langle \left[\hat{r}_T + \hat{s}_T, \hat{r}_I + \hat{s}_I, \hat{r}_M + \hat{s}_M, \hat{r}_E + \hat{s}_E\right], \left(T_{\hat{r}} + T_{\hat{s}} - T_{\hat{r}}T_{\hat{s}}, I_{\hat{r}}I_{\hat{s}}, F_{\hat{r}}F_{\hat{s}}\right)\right\rangle$$

$$\hat{r}^N \otimes \hat{s}^N = \left\langle \left[\hat{r}_T \cdot \hat{s}_T, \hat{r}_I \cdot \hat{s}_I, \hat{r}_M \cdot \hat{s}_M, \hat{r}_E \cdot \hat{s}_E\right], \left(T_{\hat{r}} \cdot T_{\hat{s}}, I_{\hat{r}} + I_{\hat{s}} - I_{\hat{r}}I_{\hat{s}}, F_{\hat{r}} + F_{\hat{s}} - F_{\hat{r}}F_{\hat{s}}\right)\right\rangle$$

$$\theta\hat{r}^N = \left\langle \left[\theta\hat{r}_T, \theta\hat{r}_I, \theta\hat{r}_M, \theta\hat{r}_E\right], \left(1-\left(1-T_{\hat{r}}\right)^{\theta}, \left(I_{\hat{r}}\right)^{\theta}, \left(F_{\hat{r}}\right)^{\theta}\right)\right\rangle$$

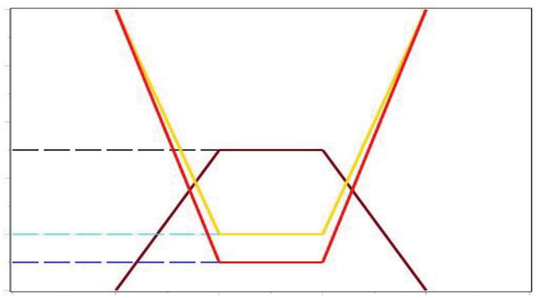

Figure 1. The graphical representation of the membership functions of TrNS.

Definition 2.4: [73]: (Comparison of any two random TrNS numbers): Let $\hat{r}^N = \left\langle \left[\hat{r}_T, \hat{r}_I, \hat{r}_M, \hat{r}_E\right], \left(T_{\hat{r}}, I_{\hat{r}}, F_{\hat{r}}\right)\right\rangle$ be a TrNS number,and then the score and accuracy function is defined, as follow:

$$s\left(\hat{r}^N\right) = \frac{1}{12}\left[\hat{r}_T + \hat{r}_I + \hat{r}_M + \hat{r}_E\right] \times \left[2 + T_{\hat{r}} - I_{\hat{r}} - F_{\hat{r}}\right]$$

$$a\left(\hat{r}^N\right) = \frac{1}{12}\left[\hat{r}_T + \hat{r}_I + \hat{r}_M + \hat{r}_E\right] \times \left[2 + T_{\hat{r}} - I_{\hat{r}} + F_{\hat{r}}\right]$$

Let $\hat{r}^N = \left\langle \left[\hat{r}_T, \hat{r}_I, \hat{r}_M, \hat{r}_E\right], \left(T_{\hat{r}}, I_{\hat{r}}, F_{\hat{r}}\right)\right\rangle$ and $\hat{s}^N = \left\langle \left[\hat{s}_T, \hat{s}_I, \hat{s}_M, \hat{s}_E\right], \left(T_{\hat{s}}, I_{\hat{s}}, F_{\hat{s}}\right)\right\rangle$ be two TrNS numbers, the ranking of $\hat{r}^N$ and $\hat{s}^N$ by score function is described as follows:

1. $if\ s\left(\hat{r}^N\right) \prec s\left(\hat{s}^N\right)\ then\ \hat{r}^N \prec \hat{s}^N$
2. $if\ s\left(\hat{r}^N\right) \approx s\left(\hat{s}^N\right)\ and\ if$
   a. $a\left(\hat{r}^N\right) \prec a\left(\hat{s}^N\right)\ then\ \hat{r}^N \prec \hat{s}^N$
   b. $a\left(\hat{r}^N\right) \succ a\left(\hat{s}^N\right)\ then\ \hat{r}^N \succ \hat{s}^N$
   c. $a\left(\hat{r}^N\right) \approx a\left(\hat{s}^N\right)\ then\ \hat{r}^N \approx \hat{s}^N$

Definition 2.5:[73]: (Comparison of any two random TNS numbers).Let $\hat{r}^{NS} = \left\langle \left[\hat{r}_T, \hat{r}_I, \hat{r}_P\right], \left(T_{\hat{r}}, I_{\hat{r}}, F_{\hat{r}}\right)\right\rangle$ be a TNS number, and then the score and accuracy functions are defined as follows:

$$s\left(\hat{r}^{NS}\right) = \frac{1}{12}\left[\hat{r}_T + 2 \cdot \hat{r}_I + \hat{r}_P\right] \times \left[2 + T_{\hat{r}} - I_{\hat{r}} - F_{\hat{r}}\right]$$

$$a\left(\hat{r}^{NS}\right) = \frac{1}{12}\left[\hat{r}_T + 2 \cdot \hat{r}_I + \hat{r}_P\right] \times \left[2 + T_{\hat{r}} - I_{\hat{r}} + F_{\hat{r}}\right]$$

Let $\hat{r}^{NS} = \left\langle \left[\hat{r}_T, \hat{r}_I, \hat{r}_P\right], \left(T_{\hat{r}}, I_{\hat{r}}, F_{\hat{r}}\right)\right\rangle$ and $\hat{s}^{NS} = \left\langle \left[\hat{s}_T, \hat{s}_I, \hat{s}_P\right], \left(T_{\hat{s}}, I_{\hat{s}}, F_{\hat{s}}\right)\right\rangle$ be two arbitrary TNSNs, the ranking of $\hat{r}^{NS}$ and $\hat{s}^{NS}$ by score function is defined as follows:

1. $if\ s\left(\hat{r}^{NS}\right) \prec s\left(\hat{s}^{NS}\right)\ then\ \hat{r}^{NS} \prec \hat{s}^{NS}$
2. $if\ s\left(\hat{r}^{NS}\right) \approx s\left(\hat{s}^{NS}\right)\ and\ if$
   a. $a\left(\hat{r}^{NS}\right) \prec a\left(\hat{s}^{NS}\right)\ then\ \hat{r}^{NS} \prec \hat{s}^{NS}$
   b. $a\left(\hat{r}^{NS}\right) \succ a\left(\hat{s}^{NS}\right)\ then\ \hat{r}^{NS} \succ \hat{s}^{NS}$
   c. $a\left(\hat{r}^{NS}\right) \approx a\left(\hat{s}^{NS}\right)\ then\ \hat{r}^{NS} \approx \hat{s}^{NS}$

Definition 2.6: [72]: Let $\hat{r} = [\hat{r}_T, \hat{r}_I, \hat{r}_M, \hat{r}_E]$ be a TrFN, and $\hat{r}_T \le \hat{r}_I \le \hat{r}_M \le \hat{r}_E$ then the centre of gravity (COG) of $\hat{r}$ can be defined as

$$COG(\hat{r}) = \begin{cases} \hat{r}, & if \quad \hat{r}_T = \hat{r}_I = \hat{r}_M = \hat{r}_E \\ \frac{1}{3}\left[\hat{r}_T + \hat{r}_I + \hat{r}_M + \hat{r}_E - \frac{\hat{r}_E\hat{r}_M - \hat{r}_I\hat{r}_T}{\hat{r}_E + \hat{r}_M - \hat{r}_I - \hat{r}_T}\right], & otherwise \end{cases}$$

Definition 2.7:[72]: (Comparison of any two random TrNS numbers).Let $\hat{s}^{NS} = \langle [\hat{s}_T, \hat{s}_I, \hat{s}_M, \hat{s}_E], (T_{\hat{s}}, I_{\hat{s}}, F_{\hat{s}}) \rangle$ be a TrNSNs,and then the score function, accuracy function, and certainty functions are defined as follows:

$$E(\hat{s}^N) = COG(\hat{r}) \times \frac{(2 + T_{\hat{s}} - I_{\hat{s}} - F_{\hat{s}})}{3},$$

$$A(\hat{s}^N) = COG(\hat{r}) \times (T_{\hat{s}} - F_{\hat{s}}),$$

$$C(\hat{s}^N) = COG(\hat{r}) \times (T_{\hat{s}})$$

Let $\hat{r}^{NS} = \langle [\hat{r}_T, \hat{r}_I, \hat{r}_P], (T_{\hat{r}}, I_{\hat{r}}, F_{\hat{r}}) \rangle$ and $\hat{s}^{NS} = \langle [\hat{s}_T, \hat{s}_I, \hat{s}_M, \hat{s}_E], (T_{\hat{s}}, I_{\hat{s}}, F_{\hat{s}}) \rangle$ be two arbitrary TrNSNs, the ranking of $\hat{r}^{NS}$ and $\hat{s}^{NS}$ by score function is defined as follows:

1. $if\ E(\hat{r}^{NS}) \succ E(\hat{s}^{NS})\ then\ \hat{r}^{NS} \succ \hat{s}^{NS}$
2. $if\ E(\hat{r}^{NS}) \approx E(\hat{s}^{NS})\ and\ if\ A(\hat{r}^{NS}) \succ A(\hat{s}^{NS}) then\ \hat{r}^{NS} \succ \hat{s}^{NS}$
3. $if\ E(\hat{r}^{NS}) \approx E(\hat{s}^{NS})\ and\ if\ A(\hat{r}^{NS}) \prec A(\hat{s}^{NS}) then\ \hat{r}^{NS} \prec \hat{s}^{NS}$
4. $if\ E(\hat{r}^{NS}) \approx E(\hat{s}^{NS})\ and\ if\ A(\hat{r}^{NS}) \prec A(\hat{s}^{NS})\ and\ C(\hat{r}^{NS}) \prec C(\hat{s}^{NS}) then\ \hat{r}^{NS} \prec \hat{s}^{NS}$
5. $if\ E(\hat{r}^{NS}) \approx E(\hat{s}^{NS})\ and\ if\ A(\hat{r}^{NS}) \approx A(\hat{s}^{NS})\ and\ C(\hat{r}^{NS}) \succ C(\hat{s}^{NS}) then\ \hat{r}^{NS} \succ \hat{s}^{NS}$
6. $if\ E(\hat{r}^{NS}) \approx E(\hat{s}^{NS})\ and\ if\ A(\hat{r}^{NS}) \approx A(\hat{s}^{NS})\ and\ C(\hat{r}^{NS}) \approx C(\hat{s}^{NS}) then\ \hat{r}^{NS} \approx \hat{s}^{NS}$

2.1 List of Abbreviation used throughout this paper.

SPP stands for "shortest path problem."
NSSPP stands for "neutrosophic shortest path problem."
NSSP stands for "neutrosophic shortest path."
NSSPL stands for "neutrosophic shortest path length."
CrSPL stands for "crisp shortest path length."
FSPP stands for "fuzzy shortest path problem."
IFSPP stands for "intuitionistic fuzzy shortest path problem."
NS stands for "neutrosophic set."
TrNSSPP stands for "trapezoidal neutrosophic shortest path."
TNSSPP stands for "triangular neutrosophic shortest path."
TrNS stands for "trapezoidal neutrosophic set."
TNS stands for "triangular neutrosophic set."
MF stands for "membership function."
TFN stands for "triangular fuzzy number."
TrFN stands for "trapezoidal fuzzy number."

3 The Proposed model

Before we start the main algorithm, we introduce a sub-section i.e., shortcoming and limitation of some of the existing models:

3.1 Discussion on shortcoming of some of the existing methods

At first, we discussed the shortcoming and limitation of the existing methods under two different type of NS environment.

Broumi et al. [68-69] first proposed a method to find the shortest path under TrNS environment. It is a very well known and propular paper in the field of neutrosophic set and system. However, the authors used some mathematical assumption to solve the problem which may be invalid in some cases. This has been discussed in detail in Example 3.1. and Example 3.2

Example 3.1**:** Broumi et al.[69]: Here authors have considered two arbitrary i.e., $\tilde{r}, \tilde{s}$ be the following TrNS numbers:

$$\tilde{r} = \langle(1,2,3,4);\ 0.4, 0.6, 0.7\rangle,$$

$$\tilde{s} = \langle(1,5,7,9);\ 0.7, 0.6, 0.8\rangle.$$

We observe that the authors used an invalid mathematical assumption to solve the problem i.e.,

$$S(\tilde{r}+\tilde{s}) = S(\tilde{r}) + S(\tilde{s})$$

Our objective is to show that above considered assumption is not valid such as

$$S(\tilde{r}+\tilde{s}) \neq S(\tilde{r}) + S(\tilde{s}).$$

Solution : According to the method of Broumi et al. [69] [see; iteration 4, page no 420, ref. Broumi et al.[69]] ,we have:

$$\tilde{r}+\tilde{s} = \langle(1,2,3,4);\ 0.4, 0.6, 0.7\rangle \oplus \langle(1,5,7,9);\ 0.7, 0.6, 0.8\rangle = \langle 2,\ 7,\ 10,\ 13);\ 0.82,\ 0.36,\ 0.56\rangle.$$

Therefore, we get, $S(\tilde{r}+\tilde{s}) = 5.06$. but $S(\tilde{r}) + S(\tilde{s}) = 3.3$.
Hence, It is clear that $S(\tilde{r}+\tilde{s}) \neq S(\tilde{r}) + S(\tilde{s})$.
Therefore, we can say that the method of Broumi et al. [68-71] is not valid. So we think there is a still a scope of improvement. So to remove this limitation we proposed our new method.

3.2. Existing crisp model in SPP
In this section, we study the notation and existing crisp SPP and proposed neutrosophic SPPs.
Notations
Ω **:** Starting node
□ **:** Final destination node
$\overline{\pi}_{mk}$: The shortest distance from an m^{th} node to k^{th} node.

$\sum_{k=1}^{s} x_{mk}$: The total flow out of node *s*.

$\sum_{k=1}^{s} x_{mk}$: The total flow into node *s*.

RK_{mk} *:* the objective cost in crisp environment
According to Bazaraa et al. [74], The crisp SPP model is as follows :

$$Min = \sum_{m=1}^{s}\sum_{k=1}^{s} RK_{mk} \cdot x_{mk}$$

Subject to: (4)

$$\sum_{m=1}^{s} x_{mk} - \sum_{k=1}^{s} x_{km} = \tilde{\underset{\sim}{\kappa}}_m$$

for all $x_{mk} \in \Re$ and non-negative where $m,k = 1,2,.....,s$ and:

$$\tilde{\underset{\sim}{\kappa}}_m = \begin{cases} 1 \ if & m = \Omega, \\ 0 \ if & m = \Omega+1, \Omega+2, ..., \square -1 \\ -1 \ if & m = \square . \end{cases} \qquad (5)$$

3.3. Transformation of crisp SPP model into nutrosophic SPP
If we replaced the parameter RK_{mk} into nutrosophic cost parameters, *i.e.* RK_{mk}^{N} , then the model is as follows:

$$Min = \sum_{m=1}^{s}\sum_{k=1}^{s} RK_{mk}^{N} . x_{mk}$$

Subject to: (6)

$$\sum_{m=1}^{s} x_{mk} - \sum_{k=1}^{s} x_{km} = \tilde{\kappa}_m \qquad m,k = 1,2,.....,s.$$

$x_{mk} \in \Re$ And are non-negative.

3.4. Algorithm: A novel approach for finding the SPP under TrNS and TNS environment

We consider a directed acyclic graph whose arc lengths are represented by neutrosophic number. Our proposed algorithm finds the shortest path from the source node *s* to the destination *d* of the graph. The steps of the algorithm are as follows:

Step 1: Let *m* be the total number of paths from *s* to *d*. Compute the score function of each arc length under the given network using the Definition 2.6-2.7.

Step 2: Find all possible paths A_i, and also find the path length of corresponding A_i, where $i = 1,2,3,......., m$,for m possible number of paths. Now, each of m paths can be considered as an arc from *s* to *d* as shown in Fig. 2. Each of these arcs are represented by a neutrosophic number.

Step 3: Calculate the summation of the score function of each arc length corresponding to the path A_i, and set that, $E(\theta_i)$ where $i = 1,2,3,......., m$.

Step 4: By ranking the score value obtained in Step 3 in ascending order, find the lowest rank which is the shortest route of the given network under neutrosophic environment.

End

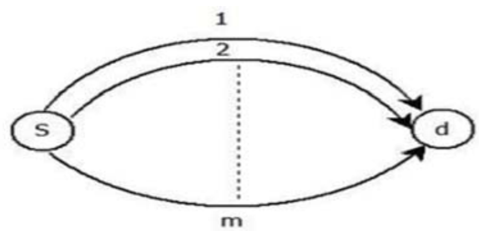

Figure. 2. m paths from source node s to destination node d are represented by m arcs

4 Example of real life application:

To justify our proposed algorithms, we consider a network shown in Fig. 3 [Broumi et al [68-71]]

Distribution network problems: In Example 4.1, and Example 4.2, we have considered a real-life problem of a distribution network for a soft drink company. Here we have considered a soft drink company which is having 6 distribution areas. This configuration is shown in Fig. 3. The time of delivery of the goods between the distribution centers can vary day to day due to many uncertain reasons such as road conjunction, driver bad health, vehicle break down, natural calamities such as flood, tsunami, earthquake and so on. Therefore, companies want to determine the range of cost per day in between the two consecutive locations but the problem is that the time is uncertain so the shortest distance will also be uncertain. This uncertainty can be avoided by predicting the shortest path using neutrosophic number and therefore we have considered TrNS and TNS numbers for our assumption where the neutrosophic cost between the two consecutive distribution centers is given in Table 1 and Table 3 respectively. The company wants to find the NSP on the basis of lowest cost of transportation for distribution between the geographical centers.

Example 4.1: Consider a network (Fig. 3), with six nodes and eight edges, where node 1 is the source node and node 6 is the destination node. The TrNS cost is given in Table 1.

Table 1. The conditions of Example 4.1.

T	H	TrNS cost	T	H	TrNS cost
1	2	<(1,2,3,4); 0.4,0.6,0.7>	3	4	<(2,4,8,9); 0.5,0.3,0.1>
1	3	<(2,5,7,8; 0.2,0.3,0.4>	3	5	<(3,4,5,10); 0.3,0.4,0.7>
2	3	<(3,7,8,9); 0.1,0.4,0.6>	4	6	<(7,8,9,10); 0.3,0.2,0.6>
2	5	<(1,5,7,9); 0.7,0.6,0.8>	5	6	<(2,4,5,7); 0.6,0.5,0.3>

Solution: Applying steps 1-4 in proposed Algorithm, we get the NSP as $1 \to 2 \to 5 \to 6$ with the lowest cost is 5.96, and the NSPL is <(4,11,15,20); 0.928, 0.18, 0.168>. It is clear that the range of NSPL is 4 to 20 and we get an optimal solution which lies inside the region. The final result is shown in Table 2.

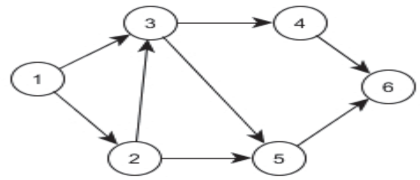

Figure 3.A network with six vertices and eight edges [Broumi et al. [68-71]]

Table 2. Final step obtained by proposed algorithm is as follows

Possible path	$E(\theta_i)$	Ranking
$A_1 : 1 \to 2 \to 5 \to 6$	5.96	1
$A_2 : 1 \to 3 \to 5 \to 6$	7.71	2
$A_3 : 1 \to 2 \to 3 \to 5 \to 6$	8.33	3
$A_4 : 1 \to 3 \to 4 \to 6$	10.97	4
$A_5 : 1 \to 2 \to 3 \to 4 \to 6$	11.59	5

Example 4.2. Consider a network from Fig. 3, with six nodes and eight edges, where node 1 is the source node and node 6 is the destination node. The TNS cost is given in Table 3.

Table 3. The conditions of Example 4.2.

T	H	TNS cost	T	H	TNS cost
1	2	<(1,2,3); 0.4,0.6,0.7>	3	4	<(2,4,8); 0.5,0.3,0.1>
1	3	<(2,5,7); 0.2,0.3,0.4>	3	5	<(3,4,5); 0.3,0.4,0.7>
2	3	<(3,7,8); 0.1,0.4,0.6>	4	6	<(7,8,9); 0.3,0.2,0.6>
2	5	<(1,5,7); 0.7,0.6,0.8>	5	6	<(2,4,5); 0.6,0.5,0.3>

Solution: Applying steps 1-4 in proposed Algorithm, the NSP is $1 \to 2 \to 5 \to 6$ and the minimum cost is 4.811; so the NSPL is < (4,11, 15); 0.93, 0.18, 0.17>. It is clear that the range of NSPL is 4 to 15 and our objective value is 4.811. So we conclude that the crisp minimum cost is 4.811. The result is shown in Table 4

Table 4. Final result of proposed Algorithm for Example 4.2 is as follows:

Possible path	$E(\theta_i)$	Ranking
$A_1 : 1 \to 2 \to 5 \to 6$	4.811	1
$A_2 : 1 \to 3 \to 5 \to 6$	6.133	2
$A_3 : 1 \to 2 \to 3 \to 5 \to 6$	6.733	3
$A_4 : 1 \to 3 \to 4 \to 6$	9.6	4
$A_5 : 1 \to 2 \to 3 \to 4 \to 6$	10.2	5

5. Result and Discussion

At first, we discussed the Example 4.1 and Example 4.2 which is considered by Broumi et al.[68-71]. We found that the proposed Algorithm gives the same shortest route as suggested by Broumi et al.[68-71]. However, our proposed methods predict the better crisp optimum cost value as compared with the mentioned existing methods.

Table 5. Logical Comparison of predicted crisp optimum cost values with the existing methods.

Example 4.1	Broumi et al. Method [68] $\succ$ our proposed method
Example 4.2	Broumi et al. Method [71] $\succ$ our proposed method

In Fig. 4 and Fig. 5 (Graphical comparison with existing methods) when we have compared our proposed method with the other existing methods, we have found that the objective value of our proposed method is smaller than to the existing methods. The best part about our proposed algorithms is that it gives the crisp optimum cost values as compared with the present existing method. This is shown in Table 5 (Logical Comparison with existing methods) and Table 6 (Numerical Comparison with existing methods) respectively. Also, we can say that the objective value obtained by our proposed algorithm lies within the neutrosophic region.
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Table 6. Numerical Comparison of our proposed method with the existing methods.

Ex	The method's name	Proposed path	SVNSPP
4.1	Method 1 [69]		NA
	Method 2 [68]	$1 \to 2 \to 5 \to 6$	Crisp optimum cost: 10.75 NSPL: < (4,11,15,20); 0.93, 0.18, 0.17>.
	Method 3 [70]	-	NA
	Method 4 [71]	-	NA
	Proposed Algorithm	$1 \to 2 \to 5 \to 6$	Suggested crisp optimum cost: 5.96 NSSPL: < (4,11,15,20); 0.93, 0.18, 0.17>.
4.2	Method 1 [69]		NA
	Method 2 [68]		NA
	Method 3 [70]		NA
	Method 4 [71]	$1 \to 2 \to 5 \to 6$	Crisp optimum cost: 8.815 NSSPL: < (4,11,15); 0.93, 0.18, 0.17>.
	Proposed Algorithm	$1 \to 2 \to 5 \to 6$	Suggested crisp optimum cost: 4.811 NSSPL: < (4,11,15); 0.93, 0.18, 0.17>. .

Because of these capabilities, we can say that our proposed algorithms are superior to the existing methods.

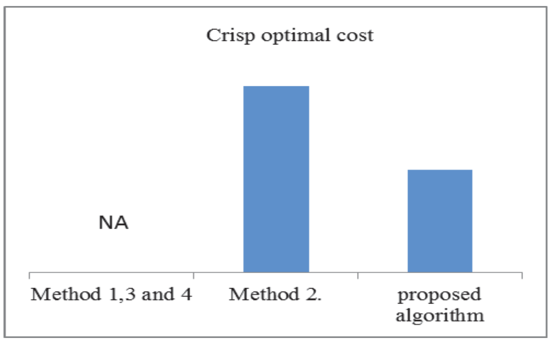

Figure 4. Comparison of crisp optimum cost value for Example 4.1 with different methods

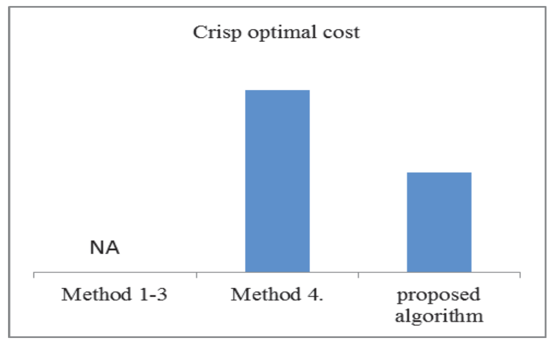

Figure 5. Comparison of crisp optimum cost values for Example 4.2 with different methods

Conclusion

In this paper, we have introduced an algorithm for solving the NSSPP. In this NSSPP, firstly we find all possible paths from source node to destination node and compute their corresponding path lengths in terms of neutrosophic number. Considering each path as an arc (from source node to destination node), we find rank of the path based on score function. The path corresponding to the lowest rank is the shortest path. An example graph is considered to demonstrate our proposed algorithm. These algorithms are not only suggested the NSSP but also able to predict the NSSPL and CrSPL. Moreover, the shortcomings of the existing algorithms are pointed out and to show the advantages of the proposed algorithms. For this purpose, we have considered NSSPP and compare with existing methods. The numerical results show that the new algorithms outperform the current methods. In the future, the proposed method can be applied to real-world problems in the field of minimum cost flow problem (MCFP), job scheduling, transportation, and so on.

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