A comment on the Collatz (3x+1) conjecture

Stephen Moore

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This note details an algorithm for calculating the solution to the Diophantine equations discussed in Winkler[1]. This note uses the terminology from that paper.

1 Discussion

It is possible and useful to derive *zero-one* sequences which we know to represent the stopping time of some integer, even if we don't know the starting integer.

If we consider a *zero-one* string which we know to represent the stopping time for some unknown integer x, we can serially apply the steps implied by the string as in the following example for the string **1110100**.

$$(\frac{3^1x+1}{2^1}, \frac{3^2x+5}{2^2}, \frac{3^3x+19}{2^3}, \frac{3^3x+19}{2^4}, \frac{3^4x+73}{2^5}, \frac{3^4x+73}{2^6}, \frac{3^4x+73}{2^7})$$

The final expression in this series will allow us to calculate what starting integer value **x** will satisfy these equations. For computational purposes, it is simpler to just assume that **x** is zero. Then the application of the zero-one string becomes:

$$(\frac{1}{2}, \frac{5}{4}, \frac{19}{8}, \frac{19}{16}, \frac{73}{32}, \frac{73}{64}, \frac{73}{128})$$

The final integer term in each expression is the same in either case.

In order to find the value of x in the example above, we only need to find a solution to the modular equation ($3^4x = 73 \, mod \, 2^7$). (Note that 73 is the numerator of the last term in the serial calculation). The modular inverse ($81^{-1} \, mod \, 128$) is 49. Therefore the solution is $((-73 * 49) \, mod \, 128)$ or 7.

This computation works the same for any *zero-one* string of any length whether or not it is a string representing a stopping time. If it is not a stopping time sequence, the computed value is the smallest integer whose full Collatz sequence has the given string as its beginning sequence. For example,: applying the computation to the string **10000001** produces a result of 213. The full Collatz sequence for 213 is **10000001000**. (In the case where an arbitrary sequence begins with zeroes, the final result must be adjusted by multiplying by 2^z, where **z** is the number of leading zeroes).

2 Summary of Algorithm

1. With a starting value of zero, serially apply the operations implied by the **zero-one** string
2. Let **f** be the numerator of the final result
3. Let **a** be the length of the sequence.
4. Let **b** be the number of **1**s in the sequence
5. let **z** be the modular inverse of $3^{\mathbf{b}} \, mod \, 2^{\mathbf{a}}$
6. Calculate $-\mathbf{f} * \mathbf{z} \, mod \, (2^{\mathbf{a}})$

The result is the value which will generate the stopping sequence.

References

[1] Mike Winkler. "The Recursive Stopping Time Structure of the 3x + 1 Function", 2018. `http://www.mikewinkler.co.nf/1709.03385_latest_update.pdf`.