Rejection of trivial objections to modal logic Ł4

Abstract: We evaluate objections to the modal logic Ł4 by six equations in contra arguments which we reject as *not* tautologous. The concluding equation invoked as (((p=p)=(q=q)))=(r=r))=((p=q)=r) is *not* tautologous. We reject the *trivial* conclusion that "modal syllogisms with both necessary premises and with mixed premises cannot be distinguished while one is necessary and another assertoric[:] Łukasiewicz' modal logic is useless for investigating Aristotelian modal syllogistic". Hence we use our VŁ4 to invalidate objections to itself.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET: ~ Not, ¬ ; + Or, ∨, ∪; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⊢; < Not Imply, less than, ∈;
= Equivalent, ≡, ⊨; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
~(y < x) (x ≤ y), (x ⊆y); (p=p) Tautology.

From: Dywan, Z. (2012). A simple axiomatization of Łukasiewicz's modal logic. Bulletin of the Section of Logic (BSL). 41:3/4, 149-153. filozof.uni.lodz.pl/bulletin/pdf/41_34_4.pdf

LET: p, q, r, s: p; q, φ; r, ψ; s.

Remark 0: Equations are numbered in order by page of text

(Lk_3) Mp → p (149.3.1)

%p>p ; NTNT NTNT NTNT NTNT (149.3.2)

(Lk_4) Mp (149.4.1)

%p=(p=p) ; CTCT CTCT CTCT CTCT (149.4.2)

(Ax_2) L(p ≡ p) (149.6.1)

#(p=p)=(p=p) ; NNNN NNNN NNNN NNNN (149.6.2)

(Ax_3) ~L(p ≡ p) (149.7.1)

~(#((p=p)=(p=p))=(p=p))=(p=p) ; CCCC CCCC CCCC CCCC (149.7.2)

M~(p ≡ p) → ~(p ≡ p) (152.1.1)

%(~((p=p)=(p=p))=(p=p))>~(p=p) ; NNNN NNNN NNNN NNNN (152.1.2)

$M{\sim}(p \equiv p)$ (152.2.1)

%(~(p=p)=(p=p))=(p=p) ; CCCC CCCC CCCC CCCC (152.2.2)

The six equations above are *not* tautologous which on their face refute the objections.

The author invokes the following equation to prove "modal syllogisms with both necessary premises and with mixed premises cannot be distinguished while one is necessary and another assertoric". (152.6.0)

$L\varphi \wedge \psi \equiv \varphi \wedge L\psi \equiv L\varphi \wedge L\psi$ (152.6.1)

((#p&q)=(p&#q))=(#p&#q) ; **FFF**N **FFF**N **FFF**N **FFF**N (152.6.2)

Remark 152.6: The respective sentences are *trivially* equivalent, but not each tautologous. The sentences so taken together as an equation can not produce a tautology based on equivalents.

Consider the form of Tautology = Tautology = Tautology. (7.1)

((p=p)=(q=q))=(r=r) ;
TTTT TTTT TTTT TTTT (7.2)

The respective sentences, while equivalent to themselves, do not constitute collegial proof of equality, as for example:

((p=(p=p))=(q=(q=q)))=(r=(r=r)) ;
FT**F**T **F**T**F**T **F**T**F**T **F**T**F**T (7.3)

Eq. 152.6.2 as rendered is *not* tautologous, and thus denies "that Łukasiewicz' modal logic is useless for investigating Aristotelian modal syllogistic".