Counting the Number of Twin Primes

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Abstract: Let P_n be the **n_th** prime. For twin primes $P_n - P_{n-1} = 2$. We give a heuristic argument that in the interval (P_n, P_n^2) as P_n gets larger, there is an increasing number of twin primes.

Let the product of the first n primes be $m=1 \text{ to } n \prod P_m = J_n$. We group the positive integers of the form $(6j - 1, 6j+1)$ in the interval $(1, J_n+1)$ into pairs. We eliminate all pairs, which contain a prime factor $P \leq P_n$. All $(6j - 1, 6j+1)$ left for which $6j < (P_{n+1})^2$ are twin primes.

$(1/6)(3/5)(5/7)\ldots((P_n - 2)/P_n)(J_n)=F_n$ is the number of pairs $(6j - 1, 6j+1)$, in $(1, J_n+1)$ with no factor less than P_{n+1} (see appendix 1). The F_n pairs are distributed approximately evenly throughout $(1, J_n+1)$. $(F_n)(P_n)^2/J_n$ is the approximate number of $(6j - 1, 6j+1)$ in the interval (P_n, P_n^2). $(F_n)(P_n)^2/J_n$ steadily increases as the P_n gets larger (see appendix 2). The table below shows the calculated versus the actual number of twin prime pairs in the interval (P_n, P_n^2)

P_n	(P_n, P_n^2)	$(F_n)(P_n)^2/J_n$	P_n	(P_n, P_n^2)	$(F_n)(P_n)^2/J_n$	P_n	(P_n, P_n^2)	$(F_n)(P_n)^2/J_n$
7	4	4	883	6581	6967	2239	32466	34920
13	9	8	1021	8450	8953	2269	33166	35735
19	17	14	1033	8613	9129	2311	34222	36845
31	30	30	1051	8881	9396	2341	34998	37711
43	50	49	1063	9062	9575	2383	36085	38845
61	91	85	1093	9518	10049	2551	40590	43800
73	123	112	1153	10408	11024	2593	41714	45113
103	208	195	1231	11667	12338	2659	43548	47188
109	223	211	1279	12476	13213	2689	44399	48043
139	320	317	1291	12659	13400	2713	45085	48724
151	374	364	1303	12862	13587	2731	45609	49264
181	492	487	1321	13183	13902	2791	47412	51192
193	547	542	1429	15103	16035	2803	47780	51523
199	574	565	1453	15527	16464	2971	52877	57090
229	716	721	1483	16065	17058	3001	53804	58171
241	772	779	1489	16164	17150	3121	57596	62344
271	927	948	1609	18426	19592	3169	59127	64114
283	988	1012	1621	18657	19812	3253	61853	67139
313	1183	1206	1669	19615	20851	3259	62052	67304
349	1419	1455	1699	20202	21531	3301	63467	68924
421	1926	1989	1723	20673	22066	3331	64422	69929
433	2034	2085	1789	22084	23547	3361	65445	71025
463	2259	2321	1873	23897	25559	3373	65827	71448
523	2760	2856	1879	24030	25669	3391	66430	72127
571	3206	3332	1933	25279	26995	3463	68872	74917
601	3486	3629	1951	25672	27444	3469	69081	75090
619	3642	3800	1999	26777	28637	3529	71177	77445
643	3881	4061	2029	27493	29357	3541	71572	77841
661	4079	4240	2083	28738	30761	3559	72180	78501
811	5732	6045	2089	28867	30880	3583	73045	79430
823	5879	6195	2113	29446	31503	3673	76213	82967
829	5951	6256	2131	29867	31982	3769	79705	86799
859	6293	6654	2143	30147	32253	3823	81702	89023

The ratio of twin prime pairs in (P_n, P_n^2) to P_n steadily increases as P_n gets larger.

ratio (P_n, P_n^2) **to** P_n			***ratio*** (P_n, P_n^2) **to** P_n			***ratio*** (P_n, P_n^2) **to** P_n		
7	4	0.57	883	6581	7.45	2239	32466	14.50
13	9	0.69	1021	8450	8.28	2269	33166	14.62
19	17	0.89	1033	8613	8.34	2311	34222	14.81
31	30	0.97	1051	8881	8.45	2341	34998	14.95
43	50	1.16	1063	9062	8.52	2383	36085	15.14
61	91	1.49	1093	9518	8.71	2551	40590	15.91
73	123	1.68	1153	10408	9.03	2593	41714	16.09
103	208	2.02	1231	11667	9.48	2659	43548	16.38
109	223	2.05	1279	12476	9.75	2689	44399	16.51
139	320	2.30	1291	12659	9.81	2713	45085	16.62
151	374	2.48	1303	12862	9.87	2731	45609	16.70
181	492	2.72	1321	13183	9.98	2791	47412	16.99
193	547	2.83	1429	15103	10.57	2803	47780	17.05
199	574	2.88	1453	15527	10.69	2971	52877	17.80
229	716	3.13	1483	16065	10.83	3001	53804	17.93
241	772	3.20	1489	16164	10.86	3121	57596	18.45
271	927	3.42	1609	18426	11.45	3169	59127	18.66
283	988	3.49	1621	18657	11.51	3253	61853	19.01
313	1183	3.78	1669	19615	11.75	3259	62052	19.04
349	1419	4.07	1699	20202	11.89	3301	63467	19.23
421	1926	4.57	1723	20673	12.00	3331	64422	19.34
433	2034	4.70	1789	22084	12.34	3361	65445	19.47
463	2259	4.88	1873	23897	12.76	3373	65827	19.52
523	2760	5.28	1879	24030	12.79	3391	66430	19.59
571	3206	5.61	1933	25279	13.08	3463	68872	19.89
601	3486	5.80	1951	25672	13.16	3469	69081	19.91
619	3642	5.88	1999	26777	13.40	3529	71177	20.17
643	3881	6.04	2029	27493	13.55	3541	71572	20.21
661	4079	6.17	2083	28738	13.80	3559	72180	20.28
811	5732	7.07	2089	28867	13.82	3583	73045	20.39
823	5879	7.14	2113	29446	13.94	3673	76213	20.75
829	5951	7.18	2131	29867	14.02	3769	79705	21.15
859	6293	7.33	2143	30147	14.07	3823	81702	21.37

Appendix 1. $(1/6)(3/5)(5/7)\ldots((P_n - 2)/P_n)(J_n)=F_n$ **is the number of pairs** $(6j - 1, 6j+1)$**,in** $(1, J_n+1)$ **with no factor less than** P_{n+1}**.** For each $(6j - 1, 6j+1)$ with all prime factors greater than P_{n-1} in $(1, J_{n-1}+1)$ there are pairs $(6j - 1+mJ_{n-1}, 6j+1+mJ_{n-1})$ for $m = 0$ *to* $P_n - 1$ in $(1, J_n+1)$. P_n and J_{n-1} are relatively prime. Thus, P_n divides $6j - 1+mJ_{n-1}$ and $6j+1+mJ_{n-1}$ each for exactly one different value of m.

Appendix 2. $(F_n)(P_n)^2/J_n$ **steadily increases as the** P_n **gets larger.** Let $P_{n+1} - P_n = a$
The ratio $((F_{n+1})(P_{n+1})^2/J_{n+1}) / ((F_n)(P_n)^2/J_n) > 1$ $(F_{n+1})(P_{n+1})^2/J_{n+1} = (F_n)(P_n+a-2)(P_n+a)^2/((J_n)(P_n+a)$
$((F_n)(P_n+a-2)(P_n+a)^2/((J_n)(P_n+a)) / ((F_n)(P_n)^2/J_n) = (P_n+a-2)(P_n+a) / P_n^2 = 1+(2a-2)/P_n+(a^2-2a)/P_n^2$

 If you wish, email comments to Jim Rock at collatz3106@gmail.com include the words **"Twin Primes"** in your subject line.