Beal Conjecture Original Proved

"5% of the people think; 10% of the people think that they think; and the other 85%would rather die than think."----Thomas Edison

"The simplest solution is usually the best solution"---Albert Einstein

Abstract

Using a direct construction approach, the author proved the original Beal conjecture that if $A^x + B^y = C^z$, where A, B, C, x, y, z are positive integers and $x, y, z > 2$, then A, B and C have a common prime factor. The proof would be complete after showing that if A and B have a common prime factor, and C^z can be produced from $A^x + B^y$. In the proof, one begins with $A^x + B^y$ and change this sum to the single power, C^z as was done in the preliminaries.
It was determined that if $A^x + B^y = C^z$, then A, B and C have a common prime factor.
The proof is very simple, and occupies a single page.

Preliminaries

$A^x + B^y = C^z$

$A = Dr$, $B = Es$, and $C = Ft$

$(Dr)^x + (Es)^y = (Ft)^z$.

Note that r, s and t are prime numbers

Case 1: Let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers. Then $A = Dr$, $B = Es$, and $C = Ft$,

If $D = 1$, $E = 1$,$F = 1$

Then, $\boxed{r^x + s^y = t^z}$ Also $A = r$, $B = s$, and $C = t$

If A, B and C have a common prime factor, then it is necessary that A and B have a common prime factor.

Example 1: $2^3 + 2^3 = 2^4 = (1 \bullet 2)^3 + (1 \bullet 2)^3 = (1 \bullet 2)^4$

One will show that another name for $2^3 + 2^3$ is 2^4.

We will write the sum on the left-hand side as a single power.

If the sum $2^3 + 2^3$ has a common prime factor, 2, then 2^4 has the common prime factor, 2.

Step 1: We will work on the two terms on the left, and change their sum to the term on the right. The two terms 2^3 and 2^3 have the common prime factor 2.. Now, if by operating on 2^3 and 2^3 together, if we obtain 2^4 , then surely 2^4 has a common prime factor as the sum of 2^3 and 2^3 since 2^4 was obtained from the sum 2^3 and 2^3, ,

Factor out the greatest common factor. $2^3 + 2^3$ $= 2^3(1+1)$ $= 2^3(2)$ $= 2^4$	It is interesting how the "(1+1)" provided the much needed 2.

Step 2: Since it has been shown above that $2^3 + 2^3 = 2^3(1+1) = 2^3(2) = 2^4$

The 2^4 must has a common factor as 2^3 and 2^3**, f**rom which it was obtained..

From above, the common prime factor is 2, and A, B and C have a common prime factor.

Therefore if $A^x + B^y = C^z$, where A, B, C, x, y, z are positive integers and $x, y, z > 2$, then A, B and C have a common prime factor.

Case 2: Let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers. Then $A = Dr$, $B = Es$, and $C = Ft$,

If $D = 1$, $E = 1$,$F \neq 1$

Then, $\boxed{r^x + s^y = (Ft)^z}$ Also $A = r$, $B = s$, and $C = Ft$

Example 2 $7^6 + 7^7 = 98^3 = (1 \bullet 7)^6 + (1 \bullet 7)^7 = (14 \bullet 7)^3$

Step 1 :We will work on the two terms on the left, and change their sum to the term on the right. Inspection shows that the two terms 7^6 and 7^7 have the common prime factor 7.. Now, if by operating on the sum $7^6 + 7^7$, we obtain 98^3,then we can conclude that all the three terms 7^6, 7^7 and 98^3 have a common prime factor, since 98^3 was obtained from the sum $7^6 + 7^7$.

Factor out the greatest common factor. $7^6 + 7^7$ $= 7^6(1 + 7)$ $= 7^6(8)$ $= 7^6(2^3)$ $= (7^2)^3(2^3)$ $= (7^2 \bullet 2)^3$ $= (49 \bullet 2)^3$ $= (98)^3$ $= 98^3$	It is interesting how the "(1+ 7)" provided the much needed 2^3.

Step 2: It has been shown that

$7^6 + 7^7 = 7^6(1 + 7) = 7^6(8) = 7^6(2^3) = (7^2)^3(2^3) = (7^2 \bullet 2)^3 = (49 \bullet 2)^3 = (98)^3$,

Since 98^3 was obtained from the sum $7^6 + 7^7$, which has a common prime factor. 7, 98^3 has the same common prime factor, 7, Therefore A, B and C have a common factor. Therefore if $A^x + B^y = C^z$, where A, B, C, x, y, z are positive integers and $x, y, z > 2$, then A, B and C have a common prime factor.

Case 3: Let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers. Then $A = Dr$, $B = Es$, and $C = Ft$,

If $D = 1$, $E \neq 1$,$F = 1$

Then, $\boxed{r^x + (Es)^y = t^z}$ Also $A = r$, $B = Es$, and $C = t$

Example 3: $3^3 + 6^3 = 3^5 = (1 \bullet 3)^3 + (2 \bullet 3)^3 = (1 \bullet 3)^5$

Step 1: We will work on the two terms on the left, and change their sum to the term on the right. Inspection shows that the two terms 3^3 and 6^3 have the common prime factor 3.. Now, if by operating on the sum $3^3 +\ 6^3$ together , we obtain 3^5 ,we can conclude that all the three terms 3^3, 6^3 and 3^5 have the common prime factor, 3 since the term on the right was produced from the two terms which have the common factor, 3

Write the sum on the left-hand side as a single power

Step 1:

Factor out the greatest common factor. $3^3 + 6^3$ $= 3^3 + (3 \bullet 2)^3$ $= 3^3 + 3^3 \bullet 2^3$ $= 3^3(1 + 2^3)$ $= 3^3(1 + 8)$ $= 3^3(9)$ $= 3^3 \bullet 3^2$ $= 3^5$	It is interesting how the "(1+ 8)" provided the much needed 3^2.

Step 2: It has been shown that

$$3^3 + 6^3 = 3^3 + (3 \bullet 2)^3 = 3^3 + 3^3 \bullet 2^3 = 3^3(1 + 2^3) = 3^3(1 + 8) = 3^3(9) = 3^3 \bullet 3^2 = 3^5,$$

$$3^3 + 6^3 = 3^5$$

From above, the common factor is 3, and A, B and C have a common factor.

Therefore if $A^x + B^y = C^z$, where A,B,C,x,y,z are positive integers and $x,y,z > 2$, then A, B and C have a common prime factor.

Case 4: Let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers. Then $A = Dr$, $B = Es$, and $C = Ft$,

If $D = 1$, $E \neq 1$,$F \neq 1$

Then, $\boxed{r^x + (Es)^y = (Ft)^z}$ Also $A = r$, $B = Es$, and $C = Ft$

Example 4 $2^9 + 8^3 = 4^5$ **=** $(1 \bullet 2)^9 + (4 \bullet 2)^3 = (2 \bullet 2)^5$

Show that $2^9 + 8^3$ equals 4^5**.**Write the sum on the left-hand side as a single power

Step 1: We will work on the two terms on the left, and change their sum to the term on the right. Inspection shows that the two terms 2^9 and 8^3 have the common prime factor 2. Now, if by operating on the sum $2^9 + 8^3$ together , we obtain 4^5, we can conclude that all the three terms 2^9, 8^3 and 4^5 have a common prime factor, since the term on the right was produced from the two terms on the left; and the two terms have a common prime factor.

Factor out the greatest common factor. $2^9 + 8^3$ $= 2^9 + (2^3)^3$ $= 2^9 + 2^9$ $= 2^9(1+1)$ $= 2^9 \bullet 2$ $= 2^{10}$ $= (2^2)^5$ $= (4)^5$ $= 4^5$	It is interesting how the "(1+ 1)" provided the much needed 2.

Step 2: It has been shown that

$2^9 + 8^3 = 2^9 + (2^3)^3 = 2^9 + 2^9 = 2^9(1+1) = 2^9 \bullet 2 = 2^{10} = (2^2)^5 = (4)^5 = 4^5$,

$2^9 + 8^3 = 4^5$

From above, the common factor is 2, and A, B and C have a common factor.

Therefore if $A^x + B^y = C^z$, where A,B,C,x,y,z are positive integers and $x,y,z > 2$, then A, B and C have a common prime factor.

Case 5: Let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers. Then $A = Dr$, $B = Es$, and $C = Ft$,

If $D \neq 1$, $E \neq 1$,$F \neq 1$

Then, $\boxed{(Dr)^x + (Es)^y = (Ft)^z}$ Also $A = Dr$, $B = Es$, and $C = Ft$

Example 5 $34^5 + 51^4 = 85^4$ **=** $(2 \bullet 17)^5 + (3 \bullet 17)^4 = (5 \bullet 17)^4$

Write the sum $34^5 + 51^4$ as a single power**.**

Step 1: We will work on the two terms on the left, and change their sum to the term on the right. Inspection shows that the two terms 34^5 and 51^4 have the common prime factor, 17.. Now, if by operating on 34^5 and 51^4 together , we obtain 85^4, we can conclude that all the three terms 34^5, 51^4 and 85^4 have a common prime factor, since the term on the right was produced from the two terms on the left with the common factor, 17. Write the sum $34^5 + 51^4$ as a single power

$(17 \bullet 2)^5 + (17 \bullet 3)^4$	
$(17^5 \bullet 2^5 + 17^4 \bullet 3^4)$	
$17^4(17 \bullet 2^5 + 1 \bullet 3^4)$	
$17^4(17 \bullet 2^5 + 3^4)$	
$= 17^4(17 \bullet 32 + 81)$	It is interesting how the
$= 17^4(625)$	$\underbrace{17 \bullet 2^5 + 3^4}_{\text{magic}}$ provided the much needed
$= 17^4(5^4)$	$625 = 5^4$
$= (17 \bullet 5)^4$	.
$= 85^4$	
Therefore, $34^5 + 51^4 = 85^4$	

Step 2: Since it has been shown that

$34^5 + 51^4 = (17 \bullet 2)^5 + (17 \bullet 3)^4$

$= 17^4(17 \bullet 2^5 + 3^4) = 17^4(17 \bullet 32 + 81) = 17^4(625) = 17^4(5^4) = (17 \bullet 5)^4 = 85^4$**,**

$34^5 + 51^4 = 85^4$**,** 85^4 was obtained from 34^5 and 51^4 which have the common prime factor , 17, A, B and C have a common factor.

Therefore if $A^x + B^y = C^z$, where A,B,C,x,y,z are positive integers and $x,y,z > 2$, then A, B and C have a common prime factor.

Example 6: Given $3^9 + 54^3 = 3^{11}$
Write the sum $3^9 + 54^3$ as a single power.

Step 1: We will operate on the two terms on the left, and change their sum to the term on the right.
Inspection shows that the two terms 3^9 and 54^3 have the common prime factor 3.. Now, if by operating on 3^9 and 54^3 together , we obtain 3^{11} ,we can conclude that all the three terms 3^9, 54^3 and 3^{11} have a common prime factor, since the term on the right was produced from the two terms on the left.

Write the sum $3^9 + 54^3$ as a single power

$3^9 + 54^3$ $= 3^9 + (9 \bullet 6)^3$ $= 3^9 + (3 \bullet 3 \bullet 3 \bullet 2)^3$ $= 3^9 + (3^3 \bullet 2)^3$ $= 3^9 + 3^9 \bullet 2^3$ $= 3^9(1 + 2^3)$ $= 3^9(1 + 8)$ $= 3^9(9)$ $= 3^9 \bullet 3^2$ $= 3^{11}$	It is interesting how the $1 + 2^3$ provided the much needed 9. .

Step 2: Since it has been shown that
$3^9 + 54^3 = 3^9 + (3 \bullet 3 \bullet 3 \bullet 2)^3 = 3^9 + 3^9 \bullet 2^3 = 3^9(1 + 2^3) = 3^9(1 + 8) = 3^9(9) = 3^9 \bullet 3^2 = 3^{11}$,
$3^9 + 54^3 = 3^{11}$, and 3^{11} was obtained from 3^9 and 54^3 which have the common prime factor , 3, A, B and C have a common prime factor..

From above, the common factor is 3, and A, B and C have a common factor.
Therefore if $A^x + B^y = C^z$, where A, B, C, x, y, z are positive integers and $x, y, z > 2$, then A, B and C have a common prime factor.

Example 7: $33^5 + 66^5 = 33^6$

Write $33^5 + 66^5$ as the single power of 33.

We will work on the two terms on the left, and change their sum to a single power

Inspection shows that the two terms 33^5 and 66^5 have the common prime factor 3. Now, if by operating on 33^5 and 66^5 together , we obtain 33^6 we can conclude that all the three terms 33^5, 66^5 and 33^6 have a common prime factor, since the term on the right was produced from the two terms on the left

Step 1: Factor the sum on the left-hand side

$33^5 + 66^5$ $= (11 \bullet 3)^5 + (11 \bullet 2 \bullet 3)^5$ $= 11^5 \bullet 3^5 + 11^5 \bullet 2^5 \bullet 3^5$ $= 11^5 \bullet 3^5(1 + 2^5)$ $= (11 \bullet 3)^5(1 + 2^5)$ $= 33^5(33)$ $= 33^6$	It is interesting how the $1 + 2^5$ provided the much needed 33

Step 2: It has been shown that

$33^5 + 66^5 = 33^5 + (33 \bullet 2)^5 = 33^5 + 33^5 \bullet 2^5 = 33^5(1 + 2^5) = 33^5(33) = 33^6$,

$33^5 + 66^5 = 33^6$

From above, there are two common prime factors, 3 and 11. and therefore, A, B and C have a common prime factor.

Therefore if $A^x + B^y = C^z$, where A, B, C, x, y, z are positive integers and $x, y, z > 2$, then A, B and C have a common prime factor.

General Proof

Given: $A^x + B^y = C^z$, A,B,C,x,y,z are positive integers and $x,y,z > 2$.
Required: To prove that A, B and C have a common prime factor.
Plan: A necessary condition for A, B and C to have a common prime factor is that A and B must have a common prime factor.
The proof would be complete after showing that If A and B have a common prime factor, and C^z has been produced from $A^x + B^y$.

Proof: Let r be a common prime factor of A and B. Then $A = Dr$, and $B = Er$., where D and E are positive integers. Also let t be a prime factor of C. Then $C = Ft$, where F is a positive integer.
On begins with $(Dr)^x + (Er)^y$ and change this sum to the single power, $C^z = (Ft)^z$ as was done in the preliminaries.

$$(Dr)^x + (Er)^y$$

$$= (Dr)^x[1 + \frac{(Es)^y}{(Dr)^x}] \qquad \text{(Factoring out the } (Dr)^x\text{)}$$

$$= (Dr)^x[\frac{(Ft)^z}{(Ft)^z} + \frac{(Es)^y}{(Dr)^x}] \qquad (\frac{(Ft)^z}{(Ft)^z} = 1, \text{ applying the substitution axiom}$$

$$= (Dr)^x[\frac{(Ft)^z(Dr)^x + (Ft)^z(Es)^y}{(Ft)^z Dr)^x}] \qquad \text{(Adding the terms within the brackets)}$$

$$= \frac{(Ft)^z(Dr)^x + (Ft)^z(Es)^y}{(Ft)^z} \qquad \text{(canceling out the } (Dr)^x\text{)}$$

$$= \frac{(Ft)^z[(Dr)^x + (Es)^y]}{(Ft)^z} \qquad \text{(Factoring out } (Ft)^z\text{)}$$

$$= \frac{(Ft)^z \bullet (Ft)^z}{(Ft)^z} \qquad (Dr)^x + (Er)^y = (Fr)^z$$

$$= (Ft)^z$$

Since $C^z = (Ft)^z$ was obtained from $A^x = (Dr)^x$ and $B^y = (Er)^y$ which have the common prime factor r, C^z also has the common prime factor, r. and one can write $(Dr)^x + (Er)^y = (Fr)^z$, where $t = r$. Therefore, A, B and C have a common prime factor.

PS

Other proofs of Beal Conjecture by the author are at viXra:1702.0331; viXra:1609.0383, viXra:1609.0157.
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