Chiral Solitons: A New Approach to Solitons Over Minkowski Space
Paul J. Werbos, Dec. 31, 2018

This letter proposes a new approach to the unmet challenge of plausibly modeling the electron and other elementary particles as “solitons,” as stable vortices of force fields. This is the only alternative in Minkowski space to the usual model of charged elementary particles as perfect point particles, with an infinite Coulomb energy of self repulsion [1], requiring that elaborate systems of renormalization must be added to the fundamental definition of any quantum field theory. Richard Feynmann has written: “The shell game that we play is technically called 'renormalization'. But no matter how clever the word, it is still what I would call a dippy process! Having to resort to such hocus-pocus has prevented us from proving that the theory of quantum electrodynamics is mathematically self-consistent. It's surprising that the theory still hasn't been proved self-consistent one way or the other by now; I suspect that renormalization is not mathematically legitimate”.[2] This letter first summarizes previous approaches using topological solitons, and then motivates and outlines the new approach.

1. Context: Long-Term Goals and Previous Approaches

This paper proposes a step towards meeting a larger unmet challenge: how can we create a plausible “soliton” model of the electron and other particles – a model which represents the electron as a set of states of continuous force fields defined over Minkowski space, with positive definite energy, and no singularity and no need for the renormalization assumptions embedded in present versions of quantum electrodynamics (QED) and the standard model of physics (EWT+QCD)?

The vast bulk of research into “solitons” in physics have studied two possible types of model: (1) “skyrrnion” models, which assume strict topological constraints on some of the fields in the model (such as use of a field required to be a unitary matrix, such that its allowed values fall on a sphere); and (2) Higgs field models, like the classic BPS monopole, which throw out the assumption that the values of fields go to zero at a distance of infinity from the center of the soliton, and use a Higgs term in the energy (Hamiltonian) to force a topological constraint. [3,4]. Many of my most recent efforts towards a soliton model of the electron have used the Higgs field mechanism, but certain recurring difficulties have kept coming up. First, the most natural models of that type do not naturally generate the striking simple symmetries of mass and lifetimes observed in modern empirical studies of particle types and masses [5]. They explain the hard quantization of electric charge, but not of intrinsic angular momentum (spin) [10]. And finally, if we had two electrons a millimeter apart in free space, would we really expect them to be surrounded by a field with topological winding number of two, as convoluted as that geometry is?

Since 2017, I have reverted to reconsidering the class of nontopological solitons which I had been studying until 1998 [4]. In 1998, the nontopological approach appeared very difficult. Now, however, it looks easier than I had thought. It also has the advantage of allowing fields without topological constraints, and allowing the usual boundary condition that fields go to zero at infinity.

In my private notes, I have mathematical and numerical ideas for a class of nontopological soliton models which I call “Ouroboros” models, including for example models based on two vector fields, A_μ and J_μ, each of which serves as a source in a way of the other. (I called them “Ouroboros” in part because it is like a snake swallowing its own tail, a strange duality, but also because of the geometry, and because the image came to me as I was looking south to Nordkapp from a ship in the Arctic Ocean.) The key numerical idea was to start from initial or asymptotic solutions centered on a thin torus (“doughnut” or “bagel” or “stellarator”), allowing the use of one-dimensional functions in r, the distance from a larger circle of radius R in space. The idea was to pursue a “ladder” of designs of that type, outlined in my notes, initially just proving that this type of stable “soliton” is possible, and building up to something which uses or reproduces the usual B and W fields of electroweak theory [6]. A google search on the term “toroidal electron” yields many serious qualitative ideas which may be useful in this approach.

This paper outlines a different approach to constructing nontopological solitons on a path leading to eventually modeling the electron and other elementary particles. In the end, it might even overlap with the Ouroboros approach! There is no fundamental contradiction that I know of.

In 2016, for a talk on Self-Organization at the mathematics department of the University of Memphis (a major site for the Erdos community), I looked at scholar.google.com for citations to “skyrmion” and to “BPS monopole.” I found 3000 of each. But as I type now, I find 15,000 citations to skyrmion, a vast explosion. It appears to me that this vast explosion was due to new research in electrical engineering, stimulated in part by a new research activity I led at NSF [7] until my retirement in 2015. But these “skyrmions” are mainly not true skyrmions; rather, they are like the “magnetic skyrmions” described in a seminal paper by Rossler et al [8], cited directly and indirectly in recent empirical work ranging from papers in Nature to technology in China. Bogdanov’s magnetic skyrmion model does create a kind of local topology, but, because it allows traditional boundary conditions and is consistent with the general approach in [4], I classify these as nontopological solitons.

Because the issues here are so important, and the approach so different from what now appears in the literature on solitons over Minkowski space, I feel it is important to make it available even before I do the kinds of analysis which I previously applied to other approaches [4,10], because at the age of 71 I cannot be sure that I will have time to do so. The family of chiral solitons (much larger than just equation 2 below) is certainly large and rich.

2. The Core New Idea

Bogdanov's energy model (equation 1 of [8]) is not a model over Minkowski space. However, it is easy enough to adopt his key idea to Minkowski space in the framework of [4].
(By the way, there is a supplement to [8], also at arxiv, which mentions gauge theory in words but does not give mathematical specifics.) In principle, we only need to expand the general framework given in section 3 of [4] by adding a sixth relativistic invariant to the list of allowed invariants given in 57:

$$\boldsymbol{f_5 \equiv \epsilon^{\mu}_{\alpha\beta\gamma}(A^{\alpha}A^{\beta}\partial_{\mu}A^{\gamma})} \qquad (1)$$

where eta is the usual completely skew-symmetric tensor of Minkowski space, the Levi-Civita tensor for Minkowski space [9]. The analogy to Bogdanov's model would have been closer if eta were only a three-tensor, as it is for three-dimensional space, but for a relativistic free space model, the Lagrangian must obey the rules for invariants over Minkowski space. There is no choice, for a theory based on one vector field, which is a sensible place to begin in exploring this class of dynamical system. Still, it is interesting that the Lagrangian for the W meson in electroweak theory also includes a field-field-derivative-field term [6].

By analogy to Bogdanov [8], we should begin by analyzing the family of systems defined by:

$$\mathcal{L} = \mathcal{L}_0 - af_5 - g(f_0) \quad , \qquad (2)$$

where $\boldsymbol{L}_0$ is the usual Lagrangian for the covariant vector field A_μ in Maxwell's Laws, where f_5 is as defined in equation 1, where f_0 is the scalar invariant $A_\mu A^\mu$ as previously defined in equations 57 of [4],
and where the choice of real parameter a and function g specifies a particular dynamical system within this family. Following Bogdanov, we refer to the middle term as the "chiral term." The first step in soliton analysis is to translate this into a Hamiltonian, equal to the well-known usual Hamiltonian of Maxwell's Laws under the Lorenz gauge plus the new terms, and then look for stable static or oscillatory [11] equilibrium states or conditions. Bogdanov asserts that the equilibria he derives for his similar model pass the basic tests of stability developed by Hobart and Derrick [24, and there is reasonable hope that this will be true here as well, for some set of choices for a and g.

The stability analysis depends on the energy density, the Hamiltonian, which in this case is simply:

$$\mathcal{H} = \frac{1}{2}(E^2 + H^2) + a\sum_{k=1}^{3}\epsilon^{k}_{\alpha\beta\gamma}A^{\alpha}A^{\beta}\frac{\partial A^{\gamma}}{\partial x_k} + g(A_0^2 - A^2) \ ,$$

(3)

where A is the three dimensional vector (A_1, A_2, A_3), where E is the gradient of A_0 , and H is the curl of A, as in ordinary treatments of Maxwell's laws. It is important to consider the possibility of stable oscillations [11] as well as static equilibrium states.

In principle, we cannot yet rule out the possibility that this by itself will yield a plausible model of the electron good enough to replicate (and enhance what is needed in quantum electrodynamics, or more.
However, it will be exciting enough if it yields any stable soliton over Minkowski space; this has never been achieved yet for any model which does not use the Skyrme or Higgs tricks, without higher order derivatives in the Lagrangian. That would be a big step forward. The tight relations between spin, charge and mass observed in nature do suggest something relatively simple and unifying underneath.

As in earlier explorations ([10] and Ouroboros), there is a natural "ladder" of possible dynamical systems within this general family, as we consider adding a second vector field and/or a two-tensor field or even spinor fields, or nonlinear functions of f_5. The key idea is to make use of the chiral term to achieve stability of the kind observed by Bogdanov. The parity violation of this class of model might even yield the curious right/left asymmetry of electroweak theory as an emergent accurate approximation to something far more elegant.

References

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