Let There Be Gravity Light
Using Only Gravitational Observations to Measure (Extract) the Speed of Light/Gravity

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Abstract

We show how one can measure the speed of gravity only using gravitational phenomena. Our approach offers several ways to measure the speed of gravity (light) and checks existing assumptions about light in new types of experiments. The speed of light is included in several well-known gravitational formulas. However, if we can measure this speed from gravitational phenomena alone, then is it the speed of light or the speed of gravity we are measuring? We think it is more than a mere coincidence that they are the same. In addition, even if it is not possible to draw strong conclusions now, our formulations support the view that there is a link between electromagnetism and gravity.

Key words: Speed of gravity, speed of light, red-shift, gravitational time dilation.

1 The Speed of Light and Gravity

The speed of light is an input factor in several gravity formulas, such as the calculation for the Schwarzschild radius

$$R_s = \frac{2GM}{c^2} \tag{1}$$

The formula can be found from Newton gravity theory [1] by setting the radius equal to where Newton's escape velocity is the speed of light, but it can also be derived from Einstein's general relativity [2] in combination with the Schwarzschild metric [3, 4] because the escape velocity there supposedly is the same [5].

Further, the Schwarzschild radius is related to gravitational time dilation

$$\frac{T_h}{T_L} = \frac{\sqrt{1-\frac{R_s}{R_h}}}{\sqrt{1-\frac{R_s}{R_L}}} = \frac{\sqrt{1-\frac{v_{e,h}^2}{c^2}}}{\sqrt{1-\frac{v_{e,L}^2}{c^2}}} \tag{2}$$

where T_h is the time as measured at a radius of R_h, T_L is the time measured at a radius of R_L from the center of the gravitational object ($R_h > R_L$), and $v_{e,L}$ and $v_{e,h}$ are the escape velocities at the two different altitudes. In other words, the speed of light is input from several gravitational phenomena that have been confirmed by observations. However, to our knowledge little has been written about working the calculation the other way around, namely using gravitational observations to measure the speed of light. This is also interesting from a fundamental point of view. Why can we find the speed of light, as we will show, simply by observing gravity phenomena? In undertaking this exploration, we will gain some insight into how the gravitational formulas were derived in the first place. The Schwarzschild radius, for example, is derived by finding the radius of a gravity object where the escape velocity is the speed of light. And when we measure the gravitational time dilation at the surface of the Earth, we can extract the speed of light from this alone.

2 Speed of Gravity/Light from a Gravitational Red-Shift Experiment

If we shoot a laser beam of light from the top of a tower to the bottom of the tower, we can measure the red-shift, with respective radii of R_h and R_L to the center of the Earth. The gravitational red-shift is then given by

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$$\frac{\bar{\lambda}_h - \bar{\lambda}_L}{\bar{\lambda}_L} = \frac{\sqrt{1 - \frac{v_{e,h}^2}{c^2}}}{\sqrt{1 - \frac{v_{e,L}^2}{c^2}}} - 1 = \frac{\sqrt{1 - \frac{2v_{o,h}^2}{c^2}}}{\sqrt{1 - \frac{2v_{o,L}^2}{c^2}}} - 1 \tag{3}$$

where v_e and v_o are the escape velocity and the orbital velocity at these two radiuses, and λ_L and λ_h are the wavelengths at the two altitudes. In a weak gravitational field, $v_o << c$ and this can be well-approximated as

$$\frac{\lambda_h - \lambda_L}{\lambda_L} \approx \frac{1 - \frac{1}{2}\frac{2v_{o,h}^2}{c^2}}{1 - \frac{\frac{1}{2}2v_{o,L}^2}{c^2}} - 1 \tag{4}$$

Solved with respect to c this gives

$$c \approx \frac{\sqrt{\lambda_h v_{o,L}^2 - \lambda_L v_{o,h}^2}}{\sqrt{\lambda_h - \lambda_L}} \tag{5}$$

And since $g = \frac{v_o^2}{R}$, we can also find the speed of the light from measuring the gravitational acceleration at the two altitudes instead of using the orbital velocities; this gives

$$c \approx \frac{\sqrt{\lambda_h g_L R_L - \lambda_L g_h R_h}}{\sqrt{\lambda_h - \lambda_L}} \tag{6}$$

If the red-shift is measured from two altitudes on Earth, then the gravitational acceleration field is naturally preferable. But we could also send a laser beam between two satellites orbiting the Earth at different altitudes and then we could just as well use the orbital velocities.

3 Speed of Gravity from Orbital Velocity and Two Atomic Clocks

If we have two atomic clocks sitting at different altitudes, then we have

$$\frac{T_h}{T_L} = \frac{\sqrt{1 - \frac{r_s}{R_h}}}{\sqrt{1 - \frac{r_s}{R_L}}} \tag{7}$$

Solved with respect to the speed of light we get

$$\begin{aligned}
\frac{T_h}{T_L} &= \frac{\sqrt{1 - \frac{\frac{2GM}{c^2}}{R_h}}}{\sqrt{1 - \frac{\frac{2GM}{c^2}}{R_L}}} \\
\frac{T_h^2}{T_L^2} &= \frac{1 - \frac{\frac{2GM}{c^2}}{R_h}}{1 - \frac{\frac{2GM}{c^2}}{R_L}} \\
\frac{T_h^2}{T_L^2} &= \frac{c^2 - \frac{2GM}{R_h}}{c^2 - \frac{2GM}{R_L}} \\
T_h^2 c^2 - \frac{2GM}{R_L} T_h^2 &= T_L^2 c^2 - \frac{2GM}{R_h} T_L^2 \\
T_h^2 c^2 - T_L^2 c^2 &= \frac{2GM}{R_L} T_h^2 - \frac{2GM}{R_h} T_L^2 \\
c &= \frac{\sqrt{2v_{o,L}^2 T_h^2 - 2v_{o,h}^2 T_L^2}}{\sqrt{T_h^2 - T_L^2}}
\end{aligned} \tag{8}$$

Obviously the Earth is rotating, so if the work is not done at the poles then we must take the different rotational speeds of the Earth at different altitudes into account (see [6] for an introduction on the topic). Therefore, we have

$$\frac{T_h}{T_L} = \frac{\sqrt{1 - \frac{r_s}{R_h} - \frac{v_{o,h}^2}{c^2}}}{\sqrt{1 - \frac{r_s}{R_L} - \frac{v_{o,L}^2}{c^2}}} \tag{9}$$

Solved with respect to the speed of light we have

$$c = \frac{\sqrt{T_L^2 v_h^2 - T_h^2 v_L^2}}{\sqrt{T_L^2 - T_h^2 + 2v_L^2 T_h^2 - 2v_h^2 T_L^2}} \tag{10}$$

The table below summarizes ways to extract the speed of light (gravity) from gravitational observations alone.

	Prediction	**Practical applicable**
From red-shift and orbital velocity	$c \approx \frac{\sqrt{\lambda_h v_{o,L}^2 - \lambda_L v_{o,h}^2}}{\sqrt{\lambda_h - \lambda_L}}$	Easy enough
From red-shift and acceleration field	$c \approx \frac{\sqrt{\lambda_h g_L R_L - \lambda_L g_h R_h}}{\sqrt{\lambda_h - \lambda_L}}$	Easy enough
Atomic clocks + orbital velocity	$c = \frac{\sqrt{2v_{o,L}^2 T_h^2 - 2v_{o,h}^2 T_L^2}}{\sqrt{T_h^2 - T_L^2}}$	Does not take rotation of Earth into account
Atomic clocks + acceleration field	$c = \frac{\sqrt{2g_L R_L T_h^2 - 2g_h R_h T_L^2}}{\sqrt{T_h^2 - T_L^2}}$	Does not take rotation of Earth into account
Atomic clocks + orbital velocity	$c = \frac{\sqrt{T_L^2 v_{o,h}^2 - T_h^2 v_{o,L}^2}}{\sqrt{T_L^2 - T_h^2 + 2v_{o,L}^2 T_h^2 - 2v_{o,h}^2 T_L^2}}$	Easy enough

Table 1: Ways to measure (extract) the speed of light/gravity from gravitational observations

Figure 1 illustrates some of the ways we can find the speed of light (gravity?) by only doing gravity observations. In Figure 1 a) we have two orbiting satellites, we send light between them and measure the red-shift. In addition, we measure the orbital velocity of the satellites and from this we know the speed of light. In Figure 1 b) we measure the gravitational time dilation at two altitudes, as well as the Earth's gravitational acceleration field at each altitude, and from this we know the speed of light (gravity).

4 Conclusion

We have shown how one can, in a simple way, extract the speed of light (gravity) from gravitational observations alone. Even if it only seems to consist of a rearrangement of existing formulas, the exercise and analysis can lead to new insights. We would note some things are so simple that they seem obvious when first pointed out, but they may have deeper implications and deserve further examination.

It is, at present, an open question on how we can extract the speed of light from gravitational phenomena. In our view, this indicates a link between gravity and electromagnetism, and there seems to be a renewed interest in the link between the two, see for example [7].

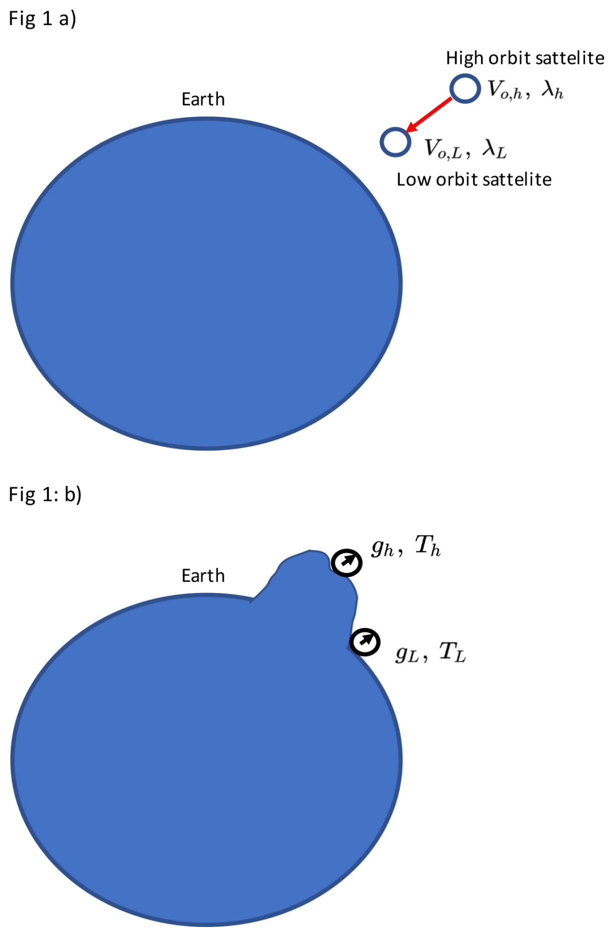

Figure 1: The figure illustrates two set-ups where we can measure the speed of light only from gravity observations. In the figure we also see what observations need to be done at each location.

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