A parametric equation of the equation $a^5 + b^5 = 2c^2$

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Abstract

The equation $a^5 + b^5 = c^2$ has no solution in integer. However, related to Fermat-Catalan conjecture, the equation $a^5 + b^5 = 2c^2$ has a solution in integer. In this article, we give a parametric equation of the equation $a^5 + b^5 = 2c^2$

$$a^5 + b^5 = 2c^2$$

The parametric equations of equation above:

$a =$

$$\frac{5s^4t^4[\frac{10s^2t^2-(5s^4+t^4)}{4}]^2-[\frac{5s^4-t^4}{4}]^4}{4}+st\frac{5s^4t^4-[\frac{10s^2t^2-(5s^4+t^4)}{4}]^2}{2}\sqrt{\frac{10s^2t^2-(5s^4+t^4)}{4}}\frac{5s^4-t^4}{4} \tag{1}$$

$b =$

$$\frac{5s^4t^4[\frac{10s^2t^2-(5s^4+t^4)}{4}]^2-[\frac{5s^4-t^4}{4}]^4}{4}-st\frac{5s^4t^4-[\frac{10s^2t^2-(5s^4+t^4)}{4}]^2}{2}\sqrt{\frac{10s^2t^2-(5s^4+t^4)}{4}}\frac{5s^4-t^4}{4} \tag{2}$$

$c = c_1c_2$

$$c_1 = \pm\sqrt{\left|\frac{5s^4t^4[\frac{10s^2t^2-(5s^4+t^4)}{4}]^2-[\frac{5s^4-t^4}{4}]^4}{4}\right|} \tag{3}$$

$$c_2 = \pm 16[\frac{5s^4t^4-[\frac{10s^2t^2-(5s^4+t^4)}{4}]^2}{4}]^4 \mp 5s^4t^4[\frac{10s^2t^2-(5t^4+s^4)}{4}]^2[\frac{5s^4-t^4}{4}]^4 \tag{4}$$

s and t are odd coprime.

The smallest solution is that $s = 1, t = 1$ then $a = 3, b = -1, c = \pm 11$
$3^5 - 1 = 2.11^2$
or $122^2 - 11^4 = 3^5$ [1]

Basing on exponent of the parametric equations above, if the equation $a^5 + b^5 = 2c^2$ have other solutions, they must be large, even very large.

Note that, the equation $\frac{10s^2t^2 - (5s^4 + t^4)}{4} = r^2$ and the equation $5x^4 - y^4 = z^2$ (general expression of c_1^2) have many infinite solutions in integer.
Ex.

For $\frac{10s^2t^2 - (5s^4 + t^4)}{4} = r^2$
s = 5, t = 7, r = 41

For $5x^4 - y^4 = z^2$
x = 13, y = 11, z = 358

Related to the Fermat - Catalan conjecture, the equations below .

$a^5 + b^5 = c^2$ has no solution in integer.

$a^8 - b^2 = c^3$ $(a^2 - b^8 = c^3)$ has no solution in integer with c odd .

If c is even , it is known : $30042907^2 - 43^8 = 96222^3$ [1]

References

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