Topological Skyrme Model with Wess-Zumino Anomaly term has Colour dependence in Quark charges and indicates Incompleteness of the pure Skyrme Model

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Abstract

The topological Skyrme has been actively studied in recent times (e.g. see Manton and Sutcliffe, Topological Solitons, Cam U Press, 2004) to understand the structure of the nucleons and the nucleus. Here through a consistent study of the electric charge, it is shown that just the Skyrme lagrangian by itself, gives charges as, $Q_p = \frac{1}{2} \, and \, Q_n = -\frac{1}{2}$; shockingly missing their empirical values. This devastating problem is rectified, only by includig an extra term (not available at Skyrme's time), arising from the Wess-Zumino anomaly. One then obtains $Q_u = \frac{2}{3} \, and \, Q_d = -\frac{1}{3}$, and thus giving the correct charges of the nucleon. It is also shown here (for the first time), that the combined Skyrme-Wess-Zumino lagrangian predicts, colour-number dependence of the electric charges as: $Q(u) = \frac{1}{2}(1 + \frac{1}{N_c})$; $Q(d) = \frac{1}{2}(-1 + \frac{1}{N_c})$ for arbitrary colour-number of the QCD group $SU(N_c)$. This gives 2/3 and -1/3 charges for $N_c = 3$. Thus it is not good enough to just have the value of charges as 2/3 and -1/3. We show that it is important to have a proper colour dependence existing within the guts of the quark charges. Though the quarks have colour built into its guts, composite protons and neutrons built up of odd-number-of-colours of quarks, turn out to be colour-free with fixed values of 1 and 0 charges, respectively (and which is good for self-consistency of QCD+QED); while the proton and neutron built up of static (colour-independent) charges 2/3 and -1/3, develop explicit colour dependence (and which is disatrous for these models).

Keywords: Topological Skyrme model, Wess-Zumino anomaly, QCD. chiral symmetry, electric charge, quark model

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The topological Skyrme model of the 1960's [1] has been focus of much activity in recent years [2-12]. The original Skyrme lagrangian needs to be supplemented with a Wess-Zumino anomaly term to ensure proper quantzation [2-6,11,12]. Our model here shall be the original Skyrme lagrangian plus the Wess-Zumino anomaly term and which we call the Skyrme-Wess-Zumino model. As well known, Wess-Zumino anomaly term is non-vanishing for three flavours and vanishes for two flavours. However, even though the Wess-Zumino anomaly term vanishes for two flavours, this two flavour reduction of the three flavour model contribution to the electric current has an interaction with electromagnetism, which was not present in the original Skyrme model. Thus one finds that just the Skyrme lagragian, by itself, fails to reproduce correct electric charge of the nucleons. We discuss how the complete Skyrme-Wess-Zumino model comes to rescue in providing the correct electric charge of three quarks [11]. As an extra benefit, we find (for the first time) that these charges in addition have colour number dependence, which had not been discovered as of now. It is well known that a certain right hypercherage quantum number $Y_R = \frac{1}{3}N_c B$ does have a colour number dependence [2-6, 11-12], in the Skyrme-Wess-Zumino model, but the colour dependence of the electric charge in the same model, has been missed out as of now. This colour dependence matches the colour dependence of electric charge in a natural extension of the structure of the Standard model. Its significance is discussed in some detail here.

Given an element U of SU(2),

$$L_\mu = U^\dagger \partial_\mu U \tag{1}$$

the Skyrme Lagrangian is given as [2-6],

$$L_S = \frac{{f_\pi}^2}{4} Tr(L_\mu L^\mu) + \frac{1}{32e^2} Tr[L_\mu, L_\nu]^2 \tag{2}$$

Here the Skyrme topological current is,

$$W_\mu = \frac{1}{24\pi^2} \epsilon_{\mu\nu\alpha\beta} Tr[L_\nu L_\alpha L_\beta] \tag{3}$$

On most general grounds this topological current is conserved, i.e. $\partial^\mu W_\mu = 0$ and giving a conserved topological charge $q = \int W_0 d^3x$. This current is independent of any WZ term that shall be added below.

Here $U(x)$ is an element of the group $SU(2)_F$,

$$U(x)^{SU(2)} = exp((i\tau^a \phi^a / f_\pi), \;\; (a = 1, 2, 3) \tag{4}$$

The solitonic structure present in the Lagrangian is obtained on making Skyrme ansatz as follows [2-6].

$$U_c(x)^{SU(2)} = exp((i/f_\pi \theta(r) \hat{r}^a \tau^a), \;\; (a = 1, 2, 3) \tag{5}$$

This $U_c(x)$ is called the Skyrmion. But on quantization, the two flavour model Skyrmion has a well known boson-fermion ambiguity [2-6]. This is rectified by going to three flavours. In that case we take,

$$U(x)^{SU(3)} = exp[\frac{i\lambda^a \phi^a(x)}{f_\pi}] \ \ (a = 1, 2..., 8) \tag{6}$$

with ϕ^a the pseudoscalar octet of π, K and η mesons. In the full topological Skyrme model we add a Wess-Zumino (WZ) effective action [2-6]

$$\Gamma_{WZ} = \frac{-i}{240\pi^2} \int_\Sigma d^5x \epsilon^{\mu\nu\alpha\beta\gamma} Tr[L_\mu L_\nu L_\alpha L_\beta L_\gamma] \tag{7}$$

on surface Σ. Thus with this anomaly term, the effective action is.

$$S_{eff} = \frac{f_\pi^2}{4} \int d^4x \ Tr \ [L_\mu L^\mu] + n \ \Gamma_{WZ} \tag{8}$$

where the winding number n is an integer $n \in Z$, the homotopy group of mapping being $\Pi_5(SU(3)) = Z$.

Write effective action as,

$$S_{eff} = \frac{f_\pi^2}{4} \int d^4x \ Tr \ [\partial_\mu U \partial^\mu U^\dagger] + n \ \Gamma_{WZ} \tag{9}$$

Taking Q as charge operator as,

$$Q = T_3 + \frac{Y}{2} \ ; \quad Y = B + S \tag{10}$$

which under a local electro-magnetic gauge transformation $h(x) = \exp(i\theta(x)Q)$ with small θ, one finds

$$\Gamma_{WZ} \to \Gamma_{WZ} - \int d^4x \partial_\mu x J^\mu(x) \tag{11}$$

where J^μ is the Noether current arising from the WZ term. This coupling to the photon field is like,

$$J_\mu = \frac{1}{48\pi^2} \epsilon^{\mu\nu\alpha\beta} Tr[Q(L_\nu L_\alpha L_\beta - R_\nu R_\alpha R_\beta)] \tag{12}$$

where $L_\mu = U^\dagger \partial_\mu U$, $R_\mu = U \partial_\mu U^\dagger$. With the electromagnetic field A_μ present, the gauge invariant form of eqn. (8) is,

$$\hat{S_{eff}} = \frac{f_\pi^2}{4} \int d^4x \ Tr \ [L_\mu L^\mu] + n \ \hat{\Gamma_{WZ}} \tag{13}$$

This means that when replacing the LHS in eqn. (10) by $\hat{\Gamma_{WZ}}$, then the RHS has two new terms involving $F_{\mu\nu}F^{\mu\nu}$. This allows us to interpret J_μ with the current carried by quarks [2-6].

With the charge operator Q (eqn. 10), T_3 does not contribute to J_μ in eqn. (12). For two flavours Y=B, and thus charge being purely isoscalar, depends upon the baryon number. To obtain the baryon current from eqn. (12), one replaces Q by $\frac{1}{N_c}$ (where N_c is the number of colours in $SU(N_c)$ - QCD for arbitrary number of colours), which is the baryon charge carried by each quark making up the baryon. For total antisymmetry, N_c number of quarks are needed to make up a baryon. Then $nJ_\mu \to nJ_\mu^B$ gives,

$$\begin{aligned} nJ_\mu^B(x) &= \frac{1}{48\pi^2}\left(\frac{n}{N_c}\right)\epsilon^{\mu\nu\alpha\beta}Tr[(L_\nu L_\alpha L_\beta - R_\nu R_\alpha R_\beta)] \\ &= \frac{1}{24\pi^2}\epsilon^{\mu\nu\alpha\beta}Tr[L_\nu L_\alpha L_\beta] \end{aligned} \tag{14}$$

This is the same as the topological current of Skyrme as given by eqn. (3). Thus the gauged WZ term gives rise to $J_\mu(x)$ which in turn gives the baryon charge. Thus though the WZ term Γ_{WZ} is zero for two-flavour case, but $J_\mu(x)$ still contibutes to the two-flavour case.

What is the significance of the above fact, that the Wess-Zumino term provides only isoscalar electric charge? Due to the fact that this isoscalar charge is proportional to baryon number; this brings in colour dependence.

Let us next look at the structure of the electric charge in the $SU(2)_F$ SWZ model (we shall study the three flavour case with all its subtleties, in a future paper). It has been pointed out by Balachandran et. al. [11, p. 176] that this has not been paid the attention it deserves. This because as we show below, it presents a serious challenge to the Skyrme lagrangian for two flavours. Following Balachandran et. al. [11], we define the electric charge operator in SU(2) as,

$$Q = \begin{pmatrix} q_1 & 0 \\ 0 & q_2 \end{pmatrix} \tag{15}$$

It induces the following transformation,

$$U(x) \to e^{i\epsilon_0\Lambda Q}U(x)e^{-i\epsilon_0\Lambda Q} = e^{\frac{i\epsilon_0\Lambda\tau_3(q_1-q_2)}{2}}U(x)e^{\frac{-i\epsilon_0\Lambda\tau_3(q_1-q_2)}{2}} \tag{16}$$

where ϵ_0 is the electromagnetic coupling constant. The Noether current associated with the above symmetry is,

$$\frac{J_\mu^{em}}{\epsilon_0} = \frac{iF_\pi^2}{8}\,Tr\,L_\mu(Q - U^\dagger QU) - \frac{i}{8\epsilon_0^2}\,Tr\,[L_\nu, Q - U^\dagger QU][L_\mu, L_\nu] \tag{17}$$

We obtain the gauge theory by replacing

$$\partial_\mu U \to D_\mu U = \partial_\mu U - i\epsilon_0\Lambda_\mu[Q, U] \tag{18}$$

To obtain constraints on charges in eqn. (15), first expand on pion fields as,

$$J_\mu^{em} = -i\epsilon_0(q_1 - q_2)(\pi_-\partial_\mu\pi_+ - \pi_+\partial_\mu\pi_-) + ... \tag{19}$$

From pion charges one gets

$$(q_1 - q_2) = 1 \tag{20}$$

Next the charges of baryons N *and* Δ with B=1 charge on using semi-classical approximation,

$$Q = \int d^4x \; J_0^{em}(\vec{x}, t) = \epsilon_0 L_\alpha \; Tr \tau_\alpha Q \tag{21}$$

From eqn. (15) we get,

$$Q = \epsilon_0 (q_1 - q_2) L_3 \tag{22}$$

On using eqn. (20),

$$Q = \epsilon_0 L_3 \tag{23}$$

L_3 is the third component of the isospiun operator, we get (in units of ϵ_0),

$$Q(proton) = +\frac{1}{2} \;\; and \;\; Q(neutron) = -\frac{1}{2} \tag{24}$$

This is in disagreement with experiment. Thus the Skyrme Lagrangian eqn. (2), fails to provide correct electric charges to proton and neutron. This is a major conundrum as far as this model is concerned. Thus studies with only the Skyrme Lagrangian (e.g. see ref. [8,9,10] for discussion) are incomplete. Thus we should take the large number of pure Skyrme model results obtained for the nucleon and for the nuclei with a pinch of salt. Thus it appears that one has to go beyond the confines of the Skyrme lagrangian.

Thus as electric charge of proton and neutron are more than what is provided above, it needs another term. And indeed we have the additional WZ term to do the job. Again let the field U be transformed by an electric charge operator Q as, $U(x) \rightarrow e^{i\Lambda\epsilon_0 Q} U(x) e^{-i\Lambda\epsilon_0 Q}$,

Making $\Lambda = \Lambda(x)$ a local transformation the Noether current is [11]

$$J_\mu{}^{em}(x) = j_\mu{}^{em}(x) + j_\mu{}^{WZ}(x) \tag{25}$$

where the first one is the standard Skyrme term and the second is the Wess-Zumino term

$$j_\mu{}^{WZ}(x) = \frac{\epsilon_0 N_c}{48\pi^2} \epsilon_{\mu\nu\lambda\sigma} Tr V^\nu V^\lambda V^\sigma (Q + U^\dagger Q U) \tag{26}$$

Remember that even though the WZ term vanishes for two flavours, its resulting contribution to electric charge does not. This term was, of course missing in the original version of the Skyrme Lagrangian (eqn. (2)). This was the reason that Skyrme model does not give the electric charge of nucleons correctly. Also any calculation based only on just the Skyrme Lagrangian, is missing this part of the full electromagnetic effect, and thus is necessarily incomplete.

One finally obtains [11, p. 208],

$$j_\mu{}^{WZ}(x) = \frac{\epsilon_0}{2} (q_1 + q_2) N_c J_\mu(x) \tag{27}$$

The WZ term correction to the electric charge is therefore,

$$\frac{\epsilon_0}{2}(q_1+q_2)N_c\int J_0(x)d^3x \tag{28}$$

Thus one gets,

$$\frac{\epsilon_0}{2}(q_1+q_2)N_cB(U_c) \tag{29}$$

With $N_c = 3$ Balachandran et. al. [11, p. 208] obtain the charges of N *and* Δ by putting,

$$q_1+q_2=\frac{1}{3} \tag{30}$$

Along with eqn. (20), they obtained [11] the charges as,

$$q_1=\frac{2}{3}\ , \quad q_2=-\frac{1}{3} \tag{31}$$

This was taken to be a success of the model [11], as it looked like the $SU(3)_F$ quark model result of Gell-Mann, wherein quark charges are given as $Q = T_3 + \frac{(Y=B+S)}{2}$ (eqn. (10)), given in terms of the two diagonal generators of the group SU(3). This was considered good as it was seen to justify the occurence of the 56 = (8,2) + (10,4) as the lowest domensional representations [2-6, 11,12], of the group $SU(6)_{FS} \supset SU(3)_F \otimes SU(2)_S$.

However as much as what eqn. (29) is actually trying to tell us, was unfortunately missed by them. Note, in eqn. (29), $N_cB(U_c) = 1$ for any colour. Due to the discussion relating to eqn. (14). and the isoscalar charge and baryon number connection, instead of a special case (eqn, 30), one obtains a general result,

$$q_1+q_2=\frac{1}{N_c} \tag{32}$$

This along with the isovector charge from the Skyrme term, eqn. 20, gives

$$Q(u)=\frac{1}{2}(1+\frac{1}{N_c});\quad Q(d)=\frac{1}{2}(-1+\frac{1}{N_c}) \tag{33}$$

This is a new result arising from the topological Skyrme-Wess-Zumino model. It is important to note that here the electric charges of quarks intrinsically depend upon colour N_c; and 2/3 and -1/3 charges are obtained only for three colours. For composite baryons to be fermions, we have to ensure that N_c is an odd number; thus $N_c = 2k+1;\ \ k = 0, 1, 2, ...$ Now assume that the proton is built up of $(k+1)$ number of u-quarks and k number of d-quarks, and vice-versa for neutron [7,13]. Thus when quark charges are colour independent, the composite baryons become colour dependent (see Appendix C). However, with the colour dependent charges of quarks, one finds that composite proton has always a charge of 1 and neutron of 0 for any colour, i.e. these are actually colour independent.

Until now every one has been using colour independent charges 2/3 and -1/3 in any study of these topological models [11, 12]. But this is wrong. We have now shown that the Skyrme-Wess-Zumino model demands colour dependent charges. In other words, what we are saying is that it is wrong to use static charges of 2/3 and -1/3 value for quarks.

Perhaps more important is the fact that these colour-dependent charges are also telling us that in the topologcal Skyrme-Wess-Zumino model, the charges are unlike the Gell-Mann $SU(3)_F$ quark model charges. Though $N_c = 3$ does give the same fractional charges as $SU(3)_F$ quark model, but they arise from different origins. How can one understand this new structure of electric charges in the topological Skyrme-Wess-Zumino model?

The Standard model of particle physics, in spite of being the most successful model in High Energy Physics, has a basic weakness, that the electric charge is arbitrary and not quantized. These are put in by hand with static values: $Q_u = \frac{2}{3}\ and\ Q_d = -\frac{1}{3}$ (see Appendix A).

However within a reaesonable extension of this model, the so called Quantized Charge Standard Model (QCSM), one obtains the same quantized and colour dependent charge of quarks as given by the Skyrme-Wess-Zumino model (see Appendix B). In Appendix C, we show how the colour dependent charges (eqn. 33) are the correct charges for consistency of QCD with QED. Hence it is not enough to have charges 2/3 and -1/3, but these should arise for $N_c = 3$ from the colour dependent charges as in eqn. (33).This is because $M(proton) \sim N_c, N_c = 3$ provides consistent understanding of the relationship between the constituent quarks and the current quarks. (see Appendix C).

Thus the colour dependenent charges as predicted by the SWZ model get unequivocal support from the Quantized Charge Standard Model [7,13].

Note that a proper incorporation of electric charge for two flavours, has major implications on the overall understanding of the structure of these models; first without the Wess-Zumino anomaly term, and the next one on incorporating it; we have arrived at two significant conclusions. Firstly, that the pure Skyrme lagrangian, as we understand it at present, is an incomplete model for providing a description of the nucleon and the nucleus. Next, the electric charge of quarks in the full Skyrme-Wess-Zumino model, has colour number dependence built into it. This shows that the structure of this model is unlike what has been popularly understood as being similar to the Gell-Mann $SU(3)_F$ quark model. Note that the quark charges in SWZ model, are supported by similar charges obtained in the Quantized Charge Stanadard Model, which is an apporpriate exension of the conventional Standard Model. What this implies for three flavours, shall be the focus of another paper in future.

Appendices:

(A). Electric Charge in the Standard Model (SM):

Glashow in studying the weak interaction in 1961, defined the electric charge in his newly proposed electro-weak (EW) group $SU(2)_W \otimes U(1)_W$. Glashow just copied the Gell-Mann-Nishijima relation of 1953 for the corresponding "strong" group $SU(2) \otimes U(1)$. Here Y_W, the weak-hypercharge, is put in by hand. Hence the electro-weak model electric charge is not quantized [7].

Glashow in 1961, was not aware of Spontaneous Symmetry Breaking (SSB) by the Englert-Brout-Higgs (EBH) mechanism, which came later in 1964, and thereafter utilized in the EW group in 1969 (Salam and Weinberg). Now the Standard Model (SM), including the strong interaction, extends the EW group structure to $SU(3)_c \otimes SU(2)_L \otimes U(1)_{Y_W}$. SM is the most successful model of particle physics at present. However the above definition of the electric charge is carried over in toto to the SM. Thus electric charge is not quantized and is arbitrary in the SM. This is a major weakness of the Standard Model [7].

Note that this unquantized charge in the Standard Model [7]: (1). Exists prior to any SSB through the EBH field; it already exists in the early universe through some unknown mechanism; (2). It is immune or independent of the strong-colour group $SU(3)_c$; (3). It is fixed with rigid values 2/3 and -1/3, i.e. no colour dependence; (4). Anomalies play no role other than being trivially satisfied by the above pre-fixed values of the hypercharge in the SM.

(B). Quantized Charge Standard Model (QCSM):

Hence we have to go beyond the above SM to get quantized charges [7]. We take the same generation structure as that in the SM and the same EBH field as an $SU(2)_L$ group doublet, $\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$.

However, major differences with respect to the above SM are [13]: (1). We start with the complete group structure $SU(N_c) \otimes SU(2)_L \otimes U(1)_{Y_W}$ *where* $N_c = 3$; (2). We do not have any arbitrarily pre-defined electric charge; (3). We take the most general definition of the electric charge in terms of generators of the above group structure as $Q = T_3 + b\,Y$ where both b and Y are unknown [13].

The first generation fermions are assigned to the following representations [13] for $SU(N_c) \otimes SU(2)_L \otimes U(1)_{Y_W}$; $q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, (N_c, 2, Y_q); u_R, (N_c, 1, Y_u)$; $d_R, (N_c, 1, Y_d); l_R = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, (1, 2, Y_l); e_R, (1, 1, Y_e)$. Five unknown hypercharges above plus the unknown Y_ϕ for EBH field (six unknown to start with).

Let the $T_3 = -\frac{1}{2}$ of the EBH field develop a nonzero vacuum expectation value $< \phi >_0$. To ensure that one of the four generators $(W_1 W_2 W_3, X)$ is thereby left unbroken (meaning that we ensure a massless photon as a generator of the $U(1)_{em}$ group), we demand: $Q < \phi >_0 = 0$. This fixes the unknown b and the electric charge now is: $Q = T_3 + (\frac{1}{2Y_\phi})Y$

For theory to be renormalisable, one ensures that all the anomalies neutralise each other for all the particles known. For each generation, cancellation of anomalies brings in the requirement of three constraints between hypercharges. Before SSB the matter particles are massless. These become massive through this process of SBB by Yukawa couplings, which due to gauge invariance yields three more relations between these unknown hypercharges. These ultimately give relations like $Y_u = Y_\phi(\frac{1}{N_c} + 1)$ (please see ref. [7,13] for details) and one obtains properly quantized electric charges in this Quantized Charge Standard

Model (which actually thus is a unified model) as [7,13],

$$Q(u) = \frac{1}{2}(1 + \frac{1}{N_c}); \quad Q(d) = \frac{1}{2}(-1 + \frac{1}{N_c}); \quad Q(\nu_e) = 0; \quad Q(e) = -1$$

Note that though $U(1)_{em}$ does not know of colour, the electric charges are actually dependent upon colour itself. Thus quark charge having colour dependence built into itself, is a significant new result of the Quantized Charge Standard Model. However this is in direct conflict with the Standard Model charges $Q(u) = \frac{2}{3}$ and $Q(d) = -\frac{1}{3}$ (i.e. indepndent of colour). Which is correct?

(C). QCD in Large Colour Limit:

The number of quarks and gluons in $SU(N_c)$ scale as $\sim N_c$, $\sim (N_c^2 - 1)$ respectively. So for large N_c, gluons dominate over quarks. The field theory of $SU(N_c)$ for large N_c reduces to a theory of weekly interacting mesons, similar to the Skyrme model where baryons arise as topological structures in a Lagrangian composed of scalar mesons only [7].

In this QCD, baryon has a finite size and has a mass going as: $M(baryon) \sim N_c$. Baryons are composed of N_c number of quarks. Composite baryons to be fermions, N_c is an odd number; thus $N_c = 2k + 1; \quad k = 0, 1, 2, ..$ respectively. Now assume that the proton is built up of $(k + 1)$ number of u-quarks and k number of d-quarks, and vice-versa for neutron [13].

Now Witten et.al. took [12] SM quark charges to be independent of colour, $Q_u = 2/3 \; and \; Q_d = -1/3$. Thus in their model the proton and neutron charges are, $Q_p = (k+1)\frac{2}{3} + k\left(-\frac{1}{3}\right) = \left(\frac{k+2}{3}\right) = \frac{N_c+3}{6}$; $Q_n = k\frac{2}{3} + (k+1)\left(-\frac{1}{3}\right) = \left(\frac{k-1}{3}\right) = \frac{N_c-3}{6}$. For arbitrary N_c, these are not even integral and actually blow up as $N_c \to \infty$. The colour dependence of proton charge is catastrophic.

Now Witten et. al [12] had unfortunately neglected the fundamental Coulomb self-energy contribution to the baryon masses. And thus the QCD plus QED contributions to baryon mass are, $M(proton) \sim N_c + C\frac{\left(\frac{N_c+3}{6}\right)^2}{R}$, where C is a constant and R is the finite size of proton. Thus the baryon mass is blowing as ${N_c}^2$, due to the QED part. This is messing up the whole analysis based on self-consistent QCD only - true for three-colours as well. This is because $M(proton) \sim N_c$ provides consistent understanding of the relationship between the constituent quarks and the current quarks in QCD for $N_C = 3$. This ${N_c}^2$ dependence is disastrous for the model of Witten et.al.[12]. Thus the definition of electric charge in the Standrd Model is inconsistent with the structure of QCD.

Next with our result of colour-dependent electric charges in the Quantized Charge Standard Model, $Q_p = (k+1)\;\frac{1}{2}\left(1 + \frac{1}{N_c}\right) + k\frac{1}{2}\left(-1 + \frac{1}{N_c}\right) = 1$; $Q_n = k\frac{1}{2}\left(1 + \frac{1}{N_c}\right) + (k+1)\;\frac{1}{2}\left(-1 + \frac{1}{N_c}\right) = 0$. Thus $Q_p = 1$, $Q_n = 0$ for arbitrary N_c - it is independent of N_c. Hence the Coulomb self-energy term of the proton remains finite. Thus the colour-dependent electric charge of the QCSM are the proper charges for quarks and proton. Hence electric charges in the QCSM are consistent with QCD while those in the SM are not [12]. For other empirical arguments supporting N_c dependence of electric charge, please see [7,13].

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