The Relationship Between CKM and PMNS Matrices in the CP-Violation Sectors

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Abstract: There are two minima in the global distribution for the mixing angle (23) in the PMNS matrix. It suggests that there are two different phenomena. We described them within the Scale-Symmetric Theory (SST). The two minima lead to two invariants for a certain relationship between the CKM and PMNS matrices in the CP-violation sectors. We also showed how SST is related to the orthodox PMNS matrix and we calculated the maximum allowed CP violation in the leptonic sector parameterized by the Jarlskog determinant – our value is 0.03291. Presented here model allows us to understand the electroweak interactions of baryons.

1. Introduction

The different size scales in Nature and the CP violation lead to the very simple initial conditions in the Scale-Symmetric Theory (SST) [1]. Such initial conditions lead to the atom-like structure of baryons and to the internal structure of bare fermions [1] which is neglected in the Standard Model and General Theory of Relativity. Bare baryons (i.e. their core) contain a torus/electric-charge/spin-1/2 and central condensate of the Einstein spacetime (ES) which is responsible for the nuclear weak interactions of baryons with coupling constant equal to $\alpha_{w(proton)} = 0.0187229$ [1]. The ratio of the poloidal speed to toroidal one of the torus leads to the Jarlskog invariant in the quark sector $J = 2.996 \cdot 10^{-5}$ – it defines the poloidal time characteristic for the weak interactions [2]. Outside the core of baryons there are the Titius-Bode (TB) orbits for the nuclear strong interactions: $R = A + dB$, where $A = 0.6974425$ fm, $B = 0.5018395$ fm, and $d = 0, 1, 2$ and 4 for the last orbit [1]. On the TB orbits can be relativistic pions. Due to the weak decays of charged pions (they orbit the baryonic core in the plane perpendicular to the spin of the torus) and the poloidal speed, the products of decay, such as the electron, make an arc equal to 270^o and are emitted along the torus's rotation axis. But we showed that the δ_{CP} phase should be broadened [2]. Emphasize that such phase can be zero or 180^o as well.

Within SST we derived the CKM matrix and PMNS matrix and showed the true origin of the mixing angles [3], [4]. The mixing angles in SST and in the orthodox matrices are the same so the two very different descriptions are equivalent. There is only one difference which causes that in the orthodox PMNS matrix, values of the elements have very low statistical significance. SST shows that $\Theta_{23}[^o] = 5\Theta_{13} = \underline{41.3250}$ [4] while experimental data lead to two minima in the global distribution: $\sin^2\Theta_{23} \approx 0.44$ i.e. $\Theta_{23}[^o] = 41.55$ which is close to the SST value, and $\sin^2\Theta_{23} \approx 0.55$ i.e. $\Theta_{23}[^o] = 47.87$ [5]. Can we explain such duality? In SST, masses of the electron-neutrino and muon-neutrino are the same so $\Delta m_{21}{}^2$ should be close to zero and it is satisfied because $\Delta m_{21}{}^2 \approx 7.5 \cdot 10^{-5}$ $(\mathrm{eV})^2$. On the other hand, mass of the tau-neutrino is three times higher but it is the electron-type neutrino [4]. This causes that the measured value $\Delta m_{32}{}^2 \approx 2.5 \cdot 10^{-3}$ $(\mathrm{eV})^2$ can be much lower than it follows from formula $\Delta m_{ij}{}^2 \equiv \Delta m_i{}^2 - \Delta m_j{}^2$. The SST value for the pure $\Delta m_{32}{}^2$ state is about 10 times higher than the experimental one but we can see that we can explain it within our model. In SST, mass of all species is 5 times higher than mass of the electron-neutrino (or muon-neutrino) – it leads to the formula $\Theta_{23} = 5\Theta_{13}$ [4]. But there are 6 different neutrinos (3 species) so the following solution is possible as well: $\Theta_{23}[^o] = 6\Theta_{13} = 49.59$ – it solves the problem of duality of Θ_{23}. Duality and broadening of the δ_{CP} phase cause that for normal ordering at 3σ range is $38.4 < \Theta_{23} < 52.8$ [5].

2. The relationship between CKM and PMNS matrices in the CP-violation sectors

First we will describe the problem by using the SST matrices [3], [4] – see Fig.1 and Fig.2 here. The CP-violation sectors we marked in red and created expressions as in the figures.

$$U_{SST}(\mathrm{PMNS}) = \begin{bmatrix} V_{e1} = 0.8294 & V_{e2} = 0.5399 & V_{e3} = -0.1438 \\ V_{\mu1} = -0.3301 & V_{\mu2} = 0.6812 & V_{\mu3} = 0.6535 \\ V_{\tau1} = 0.4507 & V_{\tau2} = -0.4945 & V_{\tau3} = 0.7432 \end{bmatrix}$$

$$U_{CP,SST}(\mathrm{PMNS}) = \left| \frac{V_{e2} V_{\mu3}}{V_{e3} V_{\mu2}} \right| = 3.602$$

Fig.1
The PMNS neutrino-mixing matrix in Scale-Symmetric Theory

$$V_{SST}(\mathrm{CKM}) = \begin{bmatrix} V_{ud} = 0.97372 & V_{us} = 0.22774 & V_{ub} = 0.00370 \\ V_{cd} = -0.22769 & V_{cs} = 0.97287 & V_{cb} = 0.04113 \\ V_{td} = 0.00587 & V_{ts} = -0.04089 & V_{tb} = 0.99915 \end{bmatrix}$$

$$V_{CP,SST}(\mathrm{CKM}) = \left| \frac{V_{us} V_{cb}}{V_{ub} V_{cs}} \right| = 2.602$$

Fig.2
The CKM quark-mixing matrix in Scale-Symmetric Theory

The PMNS-CKM invariant K is the ratio of such expressions

$$K = U_{CP,SST}(\mathrm{PMNS}) / V_{CP,SST}(\mathrm{CKM}) = 3.602 / 2.602 = 1.38\underline{4}\,, \qquad (1)$$

with 3 significant digits. On the other hand, we have [1]

$$A / B = (\alpha_{w(proton)} + \alpha_{em}) / \alpha_{w(proton)} = 1.3898 , \quad (2)$$

where $\alpha_{em} = 1/137.036$ is the fine-structure constant. Coupling constant is inversely proportional to distance so A corresponds to nuclear weak interactions while B corresponds to electroweak interactions.

From (1) and (2) follows that the invariant K for the two CP-violation sectors concerns both the atom-like structure of baryons and the electroweak interactions.

Now we can consider the orthodox matrices.

The values in the CP-violation sector in the orthodox CKM matrix are as follows [5]:

$V_{us} = 0.22534(65)$, $V_{ub} = 0.00351^{+0.00015}{}_{-0.00014}$,
$V_{cs} = 0.97344(16)$, $V_{cb} = 0.412^{+0.0011}{}_{-0.0005}$.

Applying formula from Fig.2 we obtain

$V_{CP}(CKM) = 2.717^{+0.198}{}_{-0.151}$,

so it is consistent with the SST result.

The values in the CP-violation sector in the orthodox PMNS matrix are as follows [6]:

$U_{e2} = 0.516 – 0.582$, $U_{e3} = 0.144 – 0.156$,
$U_{\mu 2} = 0.467 – 0.678$, $U_{\mu 3} = 0.639 – 0.774$.

Applying formula from Fig.1 we obtain

$U_{CP}(PMNS)$ is the interval $(3.12; 6.84)$,

so it is consistent with the SST result.

Notice that the mean values of U_{ij} lead to $U_{CP}(PMNS)^{mean} = 4.5623$ which is close to $(\pi^{+,-} - \pi^{o})/(e^{-} + e^{+})_{bare} = 4.500$ – it suggests that the electroweak interactions in the orthodox PMNS matrix with the mean values do not concern real particles (as it is in the SST matrix) – there dominate the electroweak interactions of the virtual fields around baryons [2].

We can see that there can be the second invariant

$$K^* = 1 - 1 / K = U_{CP,SST}(PMNS)^{mean} / V_{CP,SST}(CKM) = 4.500 / 2.602 = 1.73 . \quad (3)$$

This value is close to the ratio $H^{+} / Y \approx 1.72$, where H^{+} is the mass of the charged bare baryons whereas Y is the mass of central condensate which is responsible for the weak interactions.

3. The Jarlskog invariants in the quark sector and leptonic sector

In the quark sector is [7]

$$J_{CP}^{quarks} = (3.04^{+0.21}{}_{-0.20}) \cdot 10^{-5} . \quad (4)$$

It is consistent with the SST value [2].

In the leptonic sector, the Jarlskog invariant is [6]

$$J_{CP}^{leptons,max} = 0.0329 \pm 0.0007^{+0.0021}_{-0.0024} . \quad (5)$$

On the other hand, we showed that CP-violation can concern the weak masses of charged pions [2] so we can assume that following definition is correct (symbols of particles denote their masses as well)

$$J_{CP}^{leptons,max,SST} = \alpha_{w(proton)} (\pi^{+,-} - \pi^{o}) / (\alpha_{w(proton)} \pi^{+,-}) = 0.03291 . \quad (6)$$

We can see that both results are the same.

4. Summary

The mixing angles in the SST and the orthodox CKM and PMNS matrices are the same so the two very different descriptions are equivalent.

Here we described the origin of the two minima in the global distribution for the mixing angle Θ_{23} in the PMNS matrix and we showed how the Scale-Symmetric Theory couples with the PMNS matrix.

We showed that the invariant $K = 1.3898$ for the two CP-violation sectors concerns both the atom-like structure of baryons and the electroweak interactions – its value is consistent with experimental data.

We calculated value of the second invariant $K^* = 1.72$ for the two CP-violation sectors and we showed that it concerns the interior of the bare baryons i.e. it can concern the high-energy interactions.

We calculated the maximum value of the Jarlskog invariant in the leptonic sector $J_{CP}^{leptons,max,SST} = 0.03291$.

References

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