Refutation of a universal operator for interpretable deep convolution networks

Abstract: We evaluate a universal operator then apply the specified parameters to form operators for AND, OR, XOR, and MP (modus ponens). None are tautologous. This refutes the universal operator as proposed for interpretable deep convolution networks.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). For results, the 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET p, q, r, s, t, u, v, w, x, y, z:
P, ϕ, A, B, α, β, γ, b, x, y, z;
~ Not, ¬ ; + Or, ∨, ∪; - Not Or; & And, ∧, ∩; \ Not And;
\> Imply, greater than, →, ⊢, ↦; < Not Imply, less than, ∈;
= Equivalent, ≡, ⊨, :=, ⟺, ↔; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology; (z@z) **F** as contradiction, ∅, Null;
(%z<#z) C as contingency, Δ, ordinal 1;
(%z>#z) N as non-contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y).

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A universal logic operator for interpretable deep convolution networks.
arxiv.org/ftp/arxiv/papers/1901/1901.08551.pdf
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Table 1: Comparison between our proposed universal logical operator (ULO) and four classical probabilistic logical inference rules (AND, OR, XOR, MP) under independence assumption.

Note that x = P(ϕx), y = P(ϕy); MP stands for modus ponens, for which P(ϕx) = P(A); P(ϕy) = P(B|A) [ie, conditional probability meaning P(A And B)/P(A)]; and P(ϕc) = P(B).

Inference rule: U(ϕx, ϕy)	Output: P(ϕc) = P(U(ϕx, ϕy))	Logical operator parameters:	
ULO (ϕx, ϕy)	$\alpha xy + \beta y + \gamma x + b$	α, β, γ, b to be optimized	(1.0)

$$P(B)=\alpha P(A)P(B)+\beta P(B|A)+b \quad (1.1)$$

(p&s)=((t&((p&r)&(p&s)))+(((u&(p&(r&s)))\(p&r))+w)) ;
FFFF FFFF FT**F**T **F**T**F**T(2),
FFFF FFFF FT**F**T **FFFF**(1),
FFFF FFFF FT**F**T **F**T**F**T(3),

```
FFFF FFFF FTFT FFFF( 1),
FFFF FFFF FTFT FTFT( 9)
```
(1.2)

Remark 1.2: Eq. 1.2 serves as the antecedent along with the specified parameters to imply the consequent of the designated operator.

AND (ϕx, ϕy) xy $\alpha=1, \beta=0, \gamma=0, b=0$ (2.0)

Eq. 1.1 and specific parameters imply P(B)=P(A)P(B|A). (2.1)

```
(((p&s)=((t&((p&r)&(p&s)))+(((u&(p&(r&s)))\(p&r))+w)))&
(((t=(%z>#z))&(u=(z@z)))&((v=(z@z))&(w=(z@z)))))>
((p&s)=((p&r)&((u&(p&(r&s)))\(p&r)))) ;
                                TTTT TTTT TCTC TTTT( 1),
                                TTTT TTTT TNTN TTTT( 1),
                                TTTT TTTT TTTT TTTT(14)
```
(2.2)

OR (ϕx, ϕy) $x + y - xy$ $\alpha=-1, \beta=1, \gamma=1, b=0$ (3.0)

Eq. 1.1 and specific parameters imply
P(B)=P(A)+P(B|A)-P(A)P(B). (3.1)

```
((((p&s)=((t&((p&r)&(p&s)))+(((u&(p&(r&s)))\(p&r))+w)))&
(((t=(%z>#z))&(u=(z@z)))&((v=(z@z))&(w=(z@z)))))>
((p&s)=((p&r)+(((u&(p&(r&s)))\(p&r))-((p&r)&(p&s))))) ;
                                TTTT TTTT TNTN TTTT( 1),
                                TTTT TTTT TCTC TTTT( 1),
                                TTTT TTTT TTTT TTTT(14)
```
(3.2)

XOR (ϕx, ϕy) $x + y - 2xy$ $\alpha=-2, \beta=1, \gamma=1, b=0$ (4.0)

Eq. 1.1 and specific parameters imply
P(B)=P(A)+P(B)-2P(A)P(B|A). (4.1)

```
((((p&s)=((t&((p&r)&(p&s)))+(((u&(p&(r&s)))\(p&r))+w)))&
(((t=(%z>#z))&(u=(z@z)))&((v=(z@z))&(w=(z@z)))))>
((p&s)=(((p&r)+(p&s))-
((%z<#z)&((p&r)&((u&(p&(r&s)))\(p&r)))))) ;
                                TTTT TTTT TNTN TNTN( 1),
                                TTTT TTTT TCTC TCTC( 1),
                                TTTT TTTT TTTT TTTT(14)
```
(4.2)

MP (ϕx, ϕy) $xy + (1 - x)/2$ $\alpha=1, \beta=0, \gamma=-0.5, b=0.5$ (5.0)

Eq. 1.1 and specific parameters imply
P(B)=P(A)P(B|A)+(1-P(A))/2. (5.1)

```
(((p&s)=((t&((p&r)&(p&s)))+(((u&(p&(r&s)))\(p&r))+w)))&
```

```
(((t=(%z>#z))&(u=(z@z)))&((v=(z@z))&(w=(z@z)))))>
((p&s)=(((p&r)&((u&(p&(r&s)))\(p&r)))+(((%z>#z)-(p&r))\
(%z<#z)))) ;
                              TTTT TTTT TNTN TTTT( 1),
                              TTTT TTTT TTTT TTTT(15)        (5.2)
```

Eqs. 1.2-5.2 as rendered are *not* tautologous. This refutes a proposed universal operator for interpretable deep convolution networks.