Refutation of poison modal logic (PML)

Abstract: We evaluate four formulas as validities of poison modal logic (PML). None is tautologous, thereby refuting poison modal logic (PML).

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). For results, the 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET p, q, r, s: ϕ, ψ (also U), ■, ♦;
~ Not, ¬ ; + Or, ∨, ∪; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⊢, ↦, ≻, ⊃; < Not Imply, less than, ∈, ≺, ⊂;
= Equivalent, ≡, ⊨, :=, ⟺, ↔, ≙; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤; (z@z) **F** as contradiction, ∅, Null, ⊥;
(%z<#z) C as contingency, Δ, ordinal 1;
(%z>#z) N as non-contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆y); (A=B) (A~B).

From: Gross, D.; Rey, S. (2019).
Credulous acceptability, poison games and modal logic.
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Remark: Because the modal sabotage notations of ■ and ♦ act as functions, we assign them variable names.

In particular, formula

[U](p→¬♦p)∧[U](p→♦p) (2.1)

(q&(p>~(s&p)))&(q&(p>(s&p))) ;
FFT**F** **FF**T**F** **F**F T**F** **FF**T**F** (2.2)

expresses the property "the set denoted by p under function V is admissible" (in the underlying argumentation framework)

3.2 Validity and Expressivity: Examples
Fact 1. Let p ∈P and ϕ, ψ ∈Lp. The following formulas are validities of PML (w.r.t. class M∅):

¬p∧■p (3.2.3.1)

~p&(r&p) ; **FFFF FFFF FFFF FFFF** (3.2.3.2)

$$■p\leftrightarrow□p \tag{3.2.5.1}$$

(r&p)=#p ; `TCTC TNTN TCTC TNTN` (3.2.5.2)

$$□p\rightarrow(■\phi\leftrightarrow□\phi) \tag{3.2.6.1}$$

#p>((r&p)=#p) ; `TCTC TTTT TCTC TTTT` (3.2.6.2)

Proofs are omitted.

The four Eqs. above are *not* tautologous, thereby refuting poison modal logic (PML).