Refutation of the interval for model checking

Abstract: We evaluate three sub-interval relations named reflexive, proper or irreflexive, and strict. None is tautologous. This refutes those relations and model checking therefrom.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). For results, the 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. Reproducible transcripts for results are available. (See ersatz-systems.com.)

LET p, q, r, s: x, x', y, y';
~ Not, ¬ ; + Or, ∨, ∪; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⊢, ↦, ≻, ⊃; < Not Imply, less than, ∈, ≺, ⊂;
= Equivalent, ≡, ⊨, :=, ⟺, ↔, ≜; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;
(%z<#z) C as contingency, Δ, ordinal 1; (%z>#z) N as non-contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆y); (A=B) (A~B).

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arxiv.org/pdf/1901.03880.pdf, albertom.altervista.org/Th.pdf molinari.alberto@gmail.com

We consider three possible sub-interval relations: 3.1. Preliminaries

Remark 3.1: New interval operators are denoted as ⊑, ⊏, and ⊏· .
We define them here as based on connectives Not Imply < and Equivalent =.

1. [T]he reflexive sub-interval relation (denoted as ⊑), defined by [x,y]⊑[x′,y′] if and only if x′≤x and y≤y′, (3.1.1.1)

(~(p<q)&~(s<r))>~((p&r)>(q&s)) ; **F**T**FF** **F**T**F**T TTTT **F**T**FF** (3.1.1.2)

2. [T]he proper (or irreflexive) sub-interval relation (denoted as ⊏), defined by [x,y]⊏ [x′,y′] if and only if [x,y]⊑ [x′,y′] and [x,y], [x′,y′], and (3.1.2.1)

(~((q&s)>(p&r))&((p&r)@(q&s)))>((p&r)<(q&s)) ;
TTTT TTTT TT**FF** TT**F**T (3.1.2.2)

3. [T]he strict sub-interval relation (denoted as ⊏·), defined by [x,y]⊏·[x′,y′] if and only if x′<x and y<y′. (3.1.3.1)

((q<p)&(s<r))>((p&r)<(q&s)) ; TTTT TTTT TT**F**T TTTT (3.1.3.2)

Eqs. 3.1.1.2 - 3.1.3.2 as rendered are *not* tautologous. This refutes the definitions of the interval relations and hence the model checking therefrom.