Can the Twin Prime Conjecture be proven

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Abstract: Let P_n be the n_th prime. For twin primes $P_n - P_{n-1} = 2$. Let X be the number of $(6j-1, 6j+1)$ pairs in the closed interval $[P_n, P_n^2]$. The number of twin primes (TPA_n) in $[P_n, P_n^2]$ is

$$((P_n - a_n)/P_n)((P_{n-1} - a_{n-1})/P_{n-1})((P_{n-2} - a_{n-2})/P_{n-2})\ldots((5 - a_3)/5)(X).\ P_3=5,\ 1.7 < a_m < 2.3$$

For $n \geq 3$, we establish a lower bound for TPA_n $\quad (3/5)(5/7)(7/9)\ldots(P_n-2)/P_n)(X) = 3X/P_n < TPA_n$.

We exhibit a formula showing as P_n increases, the number of twin primes in the interval $[P_n, P_n^2]$ also increases.

Let $P_n - P_{n-1} = c$. For $n \geq 4$, $(TPA_{n-1})(1+(2c-2)/2P_{n-1}+(c^2-2c)/2P_{n-1}^2) < TPA_n$

Introduction:

All primes greater than or equal to five are of the form $6j-1$ or $6j+1$.

$m=1$ to n $\prod P_m = J_n$ is the product of the first n primes. For $n \geq 3$, the number of $(6j-1, 6j+1)$ pairs with no factor less than P_{n+1} in the closed interval $[1, J_n+1]$ is exactly $(1/6)(3/5)(5/7)\ldots((P_n-2)/P_n)(J_n)$.

Closely related to this is (TPA_n) the number of $(6j-1, 6j+1)$ pairs in $[P_n, P_n^2]$ with no factor less than P_{n+1}. They are all twin primes.

Let X be the number of $(6j-1, 6j+1)$ pairs in the interval $[P_n, P_n^2]$. X_m is the number of $(6j-1, 6j+1)$ pairs in the interval $[P_n, P_n^2]$ with no factor in the interval $[P_{m+1}, P_n]$. $n \geq m+1 \geq 3$. $(X_m)(P_m - a_m)/P_m = X_{m-1}$

(TPA_n) the number of twin primes in $[P_n, P_n^2]$ can be calculated (TPC_n) by the formula

$$((P_n - a_n)/P_n)((P_{n-1} - a_{n-1})/P_{n-1})((P_{n-2} - a_{n-2})/P_{n-2})\ldots((5 - a_3)/5)(X).\ P_3=5,\ 1.7 < a_m < 2.3$$

Graph 1 shows values of a_m $(P_m - a_m)$ for $[743, 743^2]$, $[3011, 3011^2]$, $[10007, 10007^2]$, $[19993, 19993^2]$

X_{pm} is the number of $(6j-1, 6j+1)$ pairs in the interval $[P_n, P_n^2]$ with a factor of P_m.
X_{pmj} is the number of $(6j-1, 6j+1)$ pairs in the interval $[P_n, P_n^2]$ with a factor of P_m that also have no factor in the interval $[P_{j+1}, P_n]$. $n \geq j+1 > m$. $(X_{pmj})(P_j - b_j)/P_j = X_{pmj-1}$

$$X_m = ((P_n - b_n)/P_n)((P_{n-1} - b_{n-1})/P_{n-1})((P_{n-2} - b_{n-2})/P_{n-2})\ldots((P_{m+1} - b_{m+1})/P_{m+1})(X_{pm})$$

H_{pmj} is $X_{pmj} - X_{pmj-1}$. For all the b_j in $(P_j - b_j)$ $n \geq j \geq m$, the weighted average b_{wm} is the sum of all the $(H_{pmj})(b_j)$ $n \geq j \geq m$ (H_{pmj} *multiplied by* b_j) divided by the sum of all the H_{pmj}, $n \geq j \geq m$.

Graph 2 show value of b_{wm} associated with each a_m of $(P_m - a_m)$.

Table 1 Let $P_n - P_{n-1} = c$. For $n \geq 4$, $(TPA_{n-1})(1+(2c-2)/2P_{n-1}+(c^2-2c)/2P_{n-1}^2) < TPA_n$

Table 2 shows the actual number of twin primes (TPA_n) versus the calculated number (TPC_n) in $[P_n, P_n^2]$ for $347 \leq P_n \leq 31153$.

Graph 3 shows how the average a_m values cycle around 2.0606 for $29917 \leq P_n \leq 31337$,

X is the number of $(6j-1, 6j+1)$ pairs in $[P_n, P_n^2]$. For $n \geq 3$, $(3/5)(5/7)(7/9)\ldots(P_n-2)/P_n)(X) = 3X/P_n < TPA_n$.

Section 1 $m=1$ to n $\prod P_m = J_n$ is the product of the first n primes.

Calculating the number of $(6j-1, 6j+1)$ pairs (F_n) with no factor $< P_{n+1}$ in $[1, J_n+1]$.

For each $(6j-1, 6j+1)$ pair with no factor less than P_n in $[1, J_{n-1}+1]$ there are pairs $(6j-1+mJ_{n-1}, 6j+1+mJ_{n-1})$ for $m = 0$ to $P_n - 1$ in $[1, J_n+1]$. P_n and J_{n-1} are relatively prime. Thus, P_n divides $6j-1+mJ_{n-1}$ and $6j+1+mJ_{n-1}$ each for exactly one different value of m.

$P_3=5,\ P_4=7.$ $\underline{F_3 = (1/6)(3/5)(J_3).}$ $F_4 = (5)(F_3).$ $J_4 = (7)(J_3)$ $F_4 / F_3 = (5/7)(J_4 / J_3).$ $\underline{F_4 = (1/6)(3/5)(5/7)(J_4).}$

The number of $(6j-1, 6j+1)$ pairs with no factor less than P_{n+1} in the interval $[1, J_n+1]$ for $n \geq 3$, is exactly

$$(1/6)(3/5)(5/7)\ldots((P_n-2)/P_n)(J_n).$$

This occurs because J_n is divisible by all primes in the interval $[P_3, P_n]$.
All the $(6j-1, 6j+1)$ pairs with no factor less than P_{n+1} in which $6j < P_{n+1}^2$ are twin primes.

Determining the number of twin primes pairs in the closed interval $[P_n, P_n^2]$.

Let X be the number of $(6j-1, 6j+1)$ pairs in the interval $[P_n, P_n^2]$. The number of twin prime pairs in $[P_n, P_n^2]$ is $((P_n - a_n)/P_n)((P_{n-1} - a_{n-1})/P_{n-1})((P_{n-2} - a_{n-2})/P_{n-2})\ldots((5 - a_3)/5)(X)$ for $n \geq m \geq 3$, $P_3=5$, $1.7 < a_m < 2.3$.
The absolute value of $P_m - 2$ for $n \geq m \geq 3$ used in calculating the number of $(6j-1, 6j+1)$ pairs in $[1, J_n+1]$ with no factor less than P_{n+1} sets a range for the values of a_m at $2 - .3 < a_m < 2 + .3$ as explained below.

Graph 1 (P_m in descending order) plots of a_m **values for** 44 **equally spaced** P_m in the intervals $[5, 743]$, $[5, 3011]$, $[5, 10007]$ and $[5, 19993]$ illustrate this formula for $[743, 743^2]$, $[3011, 3011^2]$, $[10007, 10007^2]$ and $[19993, 19993^2]$. For selected P_m they show the value of a_m $(P_m - a_m)$. Similar a_m patterns to the four plots in graph 1 are found in all sufficiently large $[P_n, P_n^2]$ intervals $(P_n > 500)$.

Graph 2 (P_m in descending order) The b_{wm} values used in graph 2 are for the same P_m, $n-1 \geq m \geq 3$ values used in graph 1. They make four similar plots in graph 2. That explains why the four plots in graph 1 are so similar. **The values of b_{wm} determines the values of a_m $(P_m - a_m)$.**

Table 2 shows the number of twin primes calculated (TPC_n) in $[P_n, P_n^2]$ for $347 \leq P_n \leq 31153$, when a_m equals 2.04, 2.06, and 2.08 for $3 \leq m \leq n$. Comparing TPC_n with TPA_n shows the average value for $P_m - a_m$ starts out near $P_m - 2.02$ for $P_n = 347$ and decreases to slightly less than $P_m - 2.06$ for $P_n = 31153$. The a_m values increase as P_n gets larger (graph 1), making the $P_m - a_m$ averages less.

For TPA_n $P_n > 30000$, the average a_m values cycle around 2.0606 **(Graph 3)**. This puts a cap (< 2.07) on the average value of a_m for all P_n.

X = the $(6j-1, 6j+1)$ pairs in $[P_n, P_n^2]$. For $n \geq 3$, $(3/5)(5/7)(7/9)\ldots(P_n-2)/P_n)(X) = 3X/P_n < TPA_n$.

$$((P_m-2)/P_m)(P_m-2)/(P_m-4) = (P_m-4)/P_m <$$

$$(P_m-2.4)/P_m < a_m/P_m, \text{ or for twin primes}$$

$$((P_m-4)/P_m)(P_m-4)/(P_m-6) = (P_m-6)/P_m <$$

$$((P_m-4.4)(P_m-2.4))/((P_m-2)(P_m)) <$$

$$((a_{m-1})(a_m))/((P_{m-1})(P_m)).$$

Thus, for $n \geq 3$, $(3/5)(5/7)(7/9)\ldots(P_n-2)/P_n)(X) = 3X/P_n < TPA_n$.

Section 2
Establishing a lower bound for the ratio TPA_n / TPA_{n-1}

Let X be the number of $(6j-1, 6j+1)$ pairs in the interval $[P_n, P_n^2]$.

The number of twin primes in the interval $[P_n, P_n^2]$ is $(a_3/5)(a_4/7)(a_5/11)\ldots(a_n/P_n)(X)$.

The average value of a_m, $3 \leq m \leq n$ can be approximated by $P_m - 2$.

X equals $(P_n^2 - P_n)/6+1$. $m=3$ to n $\prod P_m - 2 = F_n$ $m=1$ to n $\prod P_m = J_n$

The number of twin prime pairs in $[P_n, P_n^2]$ is approximately $(F_n)(P_n^2)/J_n$

TPA_n is approximately $(TPA_{n-1})((F_n)(P_n^2)/J_n) / ((F_{n-1})(P_{n-1})^2/J_{n-1})$.

TPA_n is greater than $(TPA_{n-1})((((F_n)(P_n^2)/J_n) / ((F_{n-1})(P_{n-1})^2/J_{n-1}))+1)/2$.

Calculating $\left(\left(\left((F_n)(P_n^2)/J_n\right)/\left((F_{n-1})(P_{n-1})^2/J_{n-1}\right)\right)+1\right)/2.$

Let $P_n - P_{n-1} = c.$

$$((F_n)(P_n^2)/J_n)/((F_{n-1})(P_{n-1})^2/J_{n-1}) =$$

$$\left((F_{n-1})(P_{n-1}+c-2)(P_{n-1}+c)^2/((J_{n-1})(P_{n-1}+c))\right)/\left((F_{n-1})(P_{n-1})^2/J_{n-1}\right) =$$

$$(P_{n-1}+c-2)(P_{n-1}+c)/P_{n-1}^2 =$$

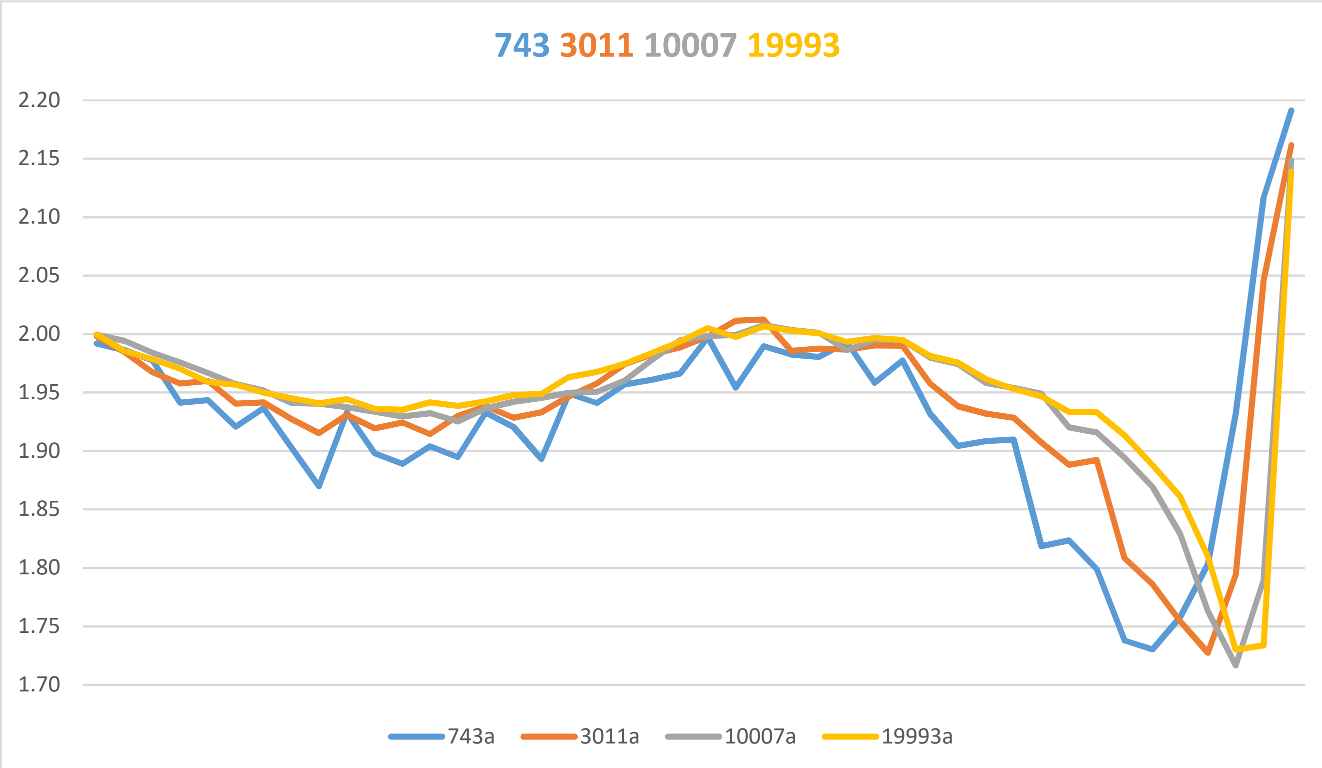
$1+(2c-2)/P_{n-1}+(c^2-2c)/P_{n-1}^2$ **Table 1 (column D) / (column C)**

For $n \geq 4,\ (TPA_{n-1})\left(1+(2c-2)/2P_{n-1}+(c^2-2c)/2P_{n-1}^2\right) < TPA_n$

Table 1 (column B)((column D/column C)+1)/2=(column F)

For $n \geq 4,\ TPA_{n-1} < TPA_n$

Graph 1 – a_m values for *44* selected P_m From *743, 3011, 10007, 19993* down to *5*



Graph 2 - b_{wm} values for *44* selected P_m From *739, 3001, 9973, 19991* down to *5*

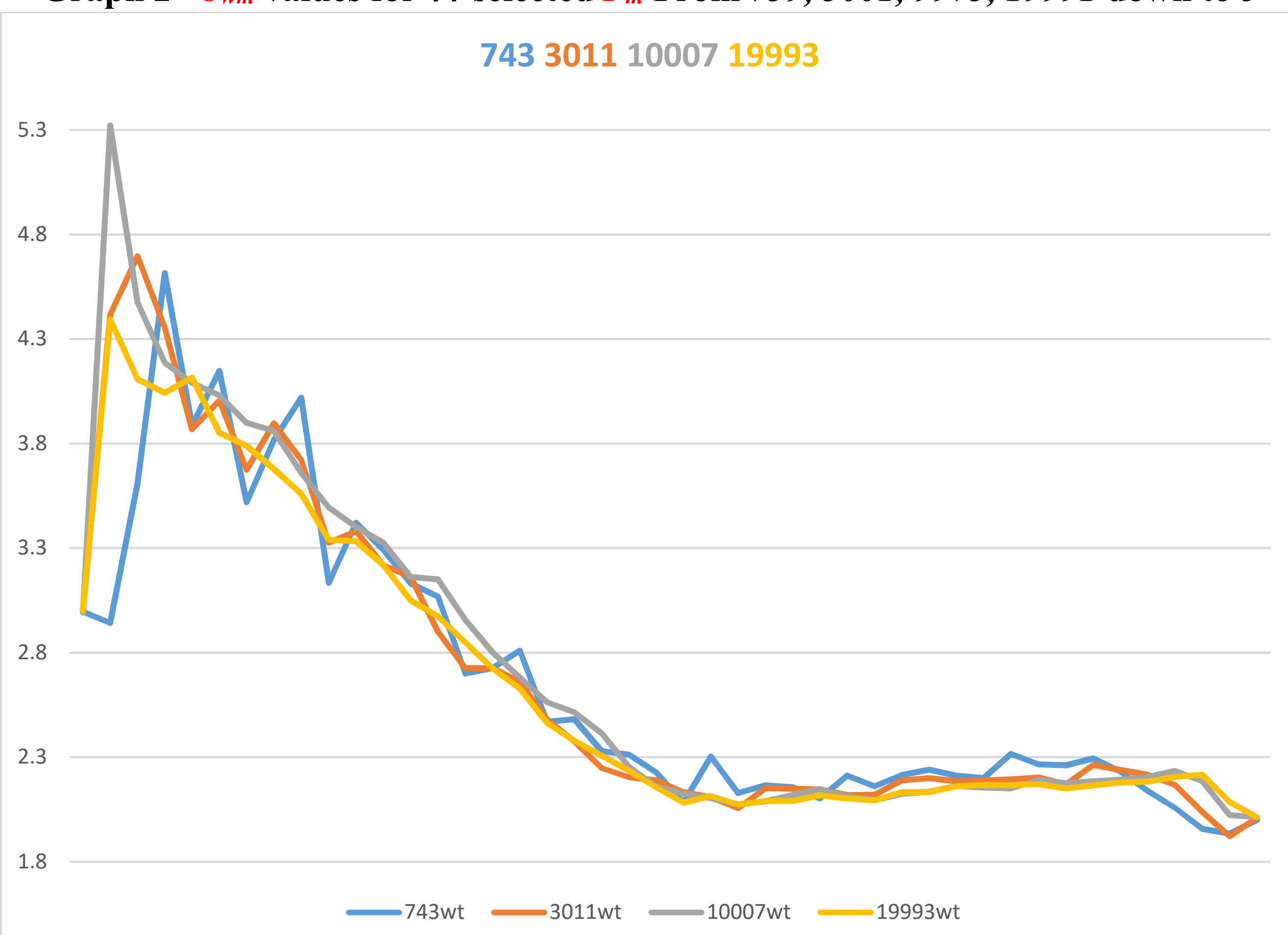


Table 1

A	B	C	D	E	F	G	H	I
prime	TPA_{n-1}	$(F_{n-1})(P_{n-1})^2/J_{n-1}$	$(F_n)(P_n)^2/J_n$	$(D/C+1)/2$	$(B)(E)$	TPA_n	F/G	B/G
71	120	109.0	112.1	1.01408	121.7	123	0.989483	0.97561
73	123	112.1	127.9	1.07047	131.7	138	0.954117	0.89130
1019	8420	8935.3	8952.8	1.00098	8428.2	8450	0.997425	0.99645
1021	8450	8952.8	9111.3	1.00885	8524.8	8586	0.992872	0.98416
2087	28819	30850.0	30879.6	1.00048	28832.8	28867	0.998816	0.99834
2089	28867	30879.6	31146.2	1.00432	28991.6	29106	0.996070	0.99179
3461	68804	74874.0	74917.3	1.00029	68823.9	68872	0.999302	0.99901
3463	68872	74917.3	75047.1	1.00087	68931.7	69019	0.998735	0.99787
4637	114316	125244.7	125298.7	1.00022	114340.6	114394	0.999534	0.99932
4639	114394	125298.7	125460.9	1.00065	114468.0	114580	0.999023	0.99838
6299	195208	215150.4	215218.7	1.00016	195239.0	195319	0.999590	0.99943
6301	195319	215218.7	215833.9	1.00143	195598.2	195879	0.998566	0.99714
8009	297317	329810.8	329893.1	1.00012	297354.1	297454	0.999664	0.99954
8011	297454	329893.1	330305.0	1.00062	297639.7	297851	0.999291	0.99867
9857	428957	476792.2	476889.0	1.00010	429000.5	429089	0.999794	0.99969
9859	429089	476889.0	477953.7	1.00112	429568.0	430004	0.998986	0.99787
11777	588001	656535.4	656646.9	1.00008	588050.9	588163	0.999809	0.99972
11779	588163	656646.9	656981.4	1.00025	588312.8	588502	0.999679	0.99942
13931	791507	885279.3	885406.4	1.00007	791563.8	791704	0.999823	0.99975
13933	791704	885406.4	889096.0	1.00208	793353.6	794778	0.998208	0.99613
16187	1033547	1158651.2	1158794.4	1.00006	1033610.9	1033796	0.999821	0.99976
16189	1033796	1158794.4	1159223.9	1.00019	1033987.6	1034307	0.999691	0.99951
18041	1254327	1408473.1	1408629.2	1.00006	1254396.5	1254586	0.999849	0.99979
18043	1254586	1408629.2	1409097.7	1.00017	1254794.6	1255094	0.999761	0.99960
20147	1527206	1717720.9	1717891.4	1.00005	1527281.8	1527479	0.999871	0.99982
20149	1527479	1717891.4	1719767.6	1.00055	1528313.1	1529106	0.999481	0.99894
21839	1763993	1985940.5	1986122.3	1.00005	1764073.7	1764289	0.999878	0.99983
21841	1764289	1986122.3	1987759.5	1.00041	1765016.2	1765719	0.999602	0.99919
23741	2047968	2308071.0	2308265.5	1.00004	2048054.3	2048281	0.999889	0.99985
23743	2048281	2308265.5	2308848.8	1.00013	2048539.8	2048899	0.999825	0.99970
26861	2555034	2883638.7	2883853.4	1.00004	2555129.1	2555371	0.999905	0.99987
26863	2555371	2883853.4	2887074.9	1.00056	2556798.3	2558027	0.999520	0.99896
28619	2861908	3233814.5	3234040.4	1.00003	2862008.0	2862279	0.999905	0.99987
28621	2862279	3234040.4	3235170.5	1.00017	2862779.1	2863372	0.999793	0.99962
31319	3365123	3806114.0	3806357.0	1.00003	3365230.4	3365489	0.999923	0.99989
31321	3365489	3806357.0	3807572.4	1.00016	3366026.3	3366653	0.999814	0.99965

Table 2 – Twin Primes in the interval [P_n, P_n^2] for $347 \leq P_n \leq 31153$

$((P_n - a_n)/P_n)((P_{n-1} - a_{n-1})/P_{n-1})((P_{n-2} - a_{n-2})/P_{n-2})\ldots((5 - a_3)/5)(X)$. For $3 \leq m \leq n$ a_m is replaced by *2.04 2.06 2.08* for $m = 3$ to n.

P_n	TPA_n	TPA_n / TPC_n 2.06	TPC_n 2.04	TPC_n 2.06	TPC_n 2.08	$3X/P_n$
347	1405	1.0637	1360.2	1320.8	1282.4	173
349	1419	1.0683	1368.0	1328.2	1289.6	174
1151	10387	1.0430	10293.6	9958.4	9633.4	575
1153	10408	1.0434	10311.2	9975.2	9649.5	576
1997	26735	1.0369	26690.2	25783.3	24905.1	998
1999	26777	1.0375	26716.5	25808.3	24929.1	999
2969	52817	1.0302	53125.5	51267.4	49470.3	1484
2971	52877	1.0307	53160.6	51300.9	49502.3	1485
3851	82712	1.0224	83885.3	80901.1	78016.6	1925
3853	82802	1.0230	83928.1	80941.9	78055.5	1926
4649	114842	1.0196	116843	112636	108572	2324
4651	114919	1.0198	116892	112683	108617	2325
5849	171367	1.0156	175132	168737	162561	2924
5851	171471	1.0159	175191	168793	162614	2925
6947	231582	1.0123	237533	228770	220312	3473
6949	231708	1.0126	237600	228834	220373	3474
8387	322646	1.0100	331826	319452	307514	4193
8389	322805	1.0103	331903	319526	307585	4194
9677	415267	1.0091	427606	411530	396024	4838
9679	415417	1.0092	427693	411613	396103	4839
10937	515723	1.0078	531882	511751	492342	5468
10939	515884	1.0079	531977	511842	492428	5469
12251	630469	1.0059	651581	626775	602859	6125
12253	630646	1.0060	651685	626874	602954	6126
13997	798218	1.0048	826113	794435	763902	6998
13999	798427	1.0049	826228	794545	764006	6999
15731	982287	1.0039	1017750	978483	940646	7865
15733	982497	1.0040	1017877	978604	940761	7866
17291	1162662	1.0026	1206394	1159636	1114583	8645
17293	1162911	1.0027	1206531	1159767	1114707	8646
18251	1280482	1.0031	1328116	1276491	1226753	9125
18253	1280728	1.0032	1328259	1276627	1226882	9126
19991	1506151	1.0015	1564964	1503866	1445014	9995
19993	1506427	1.0016	1565117	1504011	1445152	9996
21191	1671686	1.0015	1737182	1669161	1603649	10595
21193	1671950	1.0016	1737343	1669314	1603795	10596
22541	1866304	1.0009	1940784	1864560	1791151	11270
22543	1866615	1.0010	1940953	1864721	1791304	11271
23831	2061886	1.0010	2144230	2059785	1978464	11915
23833	2062203	1.0011	2144407	2059953	1978623	11916
26111	2428375	1.0000	2528479	2428472	2332181	13055
26113	2428739	1.0000	2528668	2428652	2332353	13056
27689	2697588	0.9992	2811333	2699839	2592502	13844
27691	2697935	0.9992	2811532	2700028	2592683	13845
29207	2968309	0.9994	3093224	2970220	2851812	14603
29209	2968674	0.9994	3093432	2970418	2852000	14604
31151	3333028	0.9987	3476151	3337515	3204082	15575

Graph 3 – Twin Primes in the interval [P_n, P_n^2] for $29917 \leq P_n \leq 31337$
$((P_n - a_n)/P_n)((P_{n-1} - a_{n-1})/P_{n-1})((P_{n-2} - a_{n-2})/P_{n-2})\ldots((5 - a_3)/5)(X)$. For $3 \leq m \leq n$
$a_m = 2.0606$ **for** $m = 3$ *to* n. TPA_n / TPC_n 2.0606