This occurs because J_n is divisible by all primes in the interval $[P_3, P_n]$.
All the $(6j-1, 6j+1)$ pairs with no factor less than P_{n+1} in which $6j < P_{n+1}^2$ are twin primes.

Determining the number of twin primes pairs in the closed interval $[P_n, P_n^2]$.

Let X be the number of $(6j-1, 6j+1)$ pairs in the interval $[P_n, P_n^2]$. The number of twin prime pairs in $[P_n, P_n^2]$ is $((P_n - a_n)/P_n)((P_{n-1} - a_{n-1})/P_{n-1})((P_{n-2} - a_{n-2})/P_{n-2})\ldots((5 - a_3)/5)(X)$ for $n \geq m \geq 3$, $P_3=5$, $1.7 < a_m < 2.3$.
The $P_m - 2$ for $n \geq m \geq 3$ used in calculating the number of $(6j-1, 6j+1)$ pairs in $[1, J_n+1]$ with no factor less than P_{n+1} sets a range for the values of a_m at $2 - .3 < a_m < 2 + .3$ as explained under **Graph 1** and **Graph 2**.

Graph 1 (P_m in descending order) plots of a_m **values for** 44 **equally spaced** P_m in the intervals $[5, 743]$, $[5, 3011]$, $[5, 10007]$ and $[5, 19993]$ illustrate this formula for $[743, 743^2]$, $[3011, 3011^2]$, $[10007, 10007^2]$ and $[19993, 19993^2]$. For selected P_m they show the value of a_m $(P_m - a_m)$. As explained under **Graph 2,** similar a_m patterns to the four plots in graph 1 are found in $[P_n, P_n^2]$ intervals for $(P_n > 500)$.

Graph 2 (P_m in descending order) The b_{wm} values used in graph 2 are for the same P_m, $n-1 \geq m \geq 3$ values used in graph 1. There are $(P_n^2 - P_n)/6+1=X$ $(6j-1, 6j+1)$ pairs. P_m divides the $6j-1$ and $6j+1$ at regular intervals a total of the largest integer divisor S, or $S+1$ of X/P_m times. For P_t, $m \leq t \leq n$ Largest integer divisor Y or $Y+1$ of S $(S+1)/P_t$ (of the $6j-1$ and $6j+1$ that are divisible by P_m) are divisible at regular intervals by P_t. Not all divisions are unique because the same $6j-1$ and $6j+1$ may be divisible by more than one P_t, $m \leq t \leq n$. Because all these divisions are linear, similar b_m patterns to the four plots in graph 2 are found in $[P_n, P_n^2]$ intervals for $(P_n > 500)$. **The values of b_{wm} determine the values of a_m $(P_m - a_m)$,** which explains why similar a_m patterns to the four plots in graph 1 are found in $[P_n, P_n^2]$ intervals for $(P_n > 500)$.

Table 1 shows the number of twin primes calculated (TPC_n) in $[P_n, P_n^2]$ for $347 \leq P_n \leq 31153$, when a_m equals 2.04, 2.06, and 2.08 for $3 \leq m \leq n$. Comparing TPC_n with TPA_n shows the average value for $P_m - a_m$ starts out near $P_m - 2.02$ for $P_n = 347$ and decreases to slightly less than $P_m - 2.06$ for $P_n = 31153$. The a_m values increase as P_n gets larger (graph 1), making the $P_m - a_m$ averages less.

For TPA_n $P_n > 30000$, the average a_m values cycle around 2.0606 **(Graph 3)**. This happens because similar a_m patterns to the four plots in graph 1 are found in $[P_n, P_n^2]$ intervals for $(P_n > 500)$.

This puts a cap (< 2.07) on the average value of a_m for all P_n.

X = the $(6j-1, 6j+1)$ pairs in $[P_n, P_n^2]$. For $n \geq 3$, $(3/5)(5/7)(7/9)\ldots(P_n-2)/P_n)(X) = 3X/P_n < TPA_n$.

$$((P_m - 4)/P_m - 2))(P_m - 2)/(P_m) = (P_m - 4)/P_m <$$

$$(P_m - 2.3)/P_m < (P_m - a_m)/P_m, \text{ or for twin primes}$$

$$((P_m - 6)/(P_m - 4))(P_m - 4)/P_m) = (P_m - 6)/P_m <$$

$$((P_m - 4.3)/(P_m - 2))(P_m - 2.3)/P_m < ((P_{m-1} - a_{m-1})/P_{m-1})(P_m - a_m)/P_m$$

Thus, for $n \geq 3$, $(3/5)(5/7)(7/9)\ldots(P_n-2)/P_n)(X) = 3X/P_n < TPA_n$.

Section 2

Establishing a lower bound for the ratio TPA_n/TPA_{n-1}

Let X be the number of $(6j-1, 6j+1)$ pairs in the interval $[P_n, P_n^2]$.

The number of twin primes in the interval $[P_n, P_n^2]$ is $(a_3/5)(a_4/7)(a_5/11)\ldots(a_n/P_n)(X)$.

The average value of a_m, $3 \leq m \leq n$ can be approximated by $P_m - 2$.

X equals $(P_n^2 - P_n)/6+1$. $m=3 \text{ to } n \prod P_m - 2 = F_n$ $m=1 \text{ to } n \prod P_m = J_n$

The number of twin prime pairs in $[P_n, P_n^2]$ is approximately $(F_n)(P_n^2)/J_n$

TPA_n is approximately $(TPA_{n-1})((F_n)(P_n^2)/J_n) / ((F_{n-1})(P_{n-1})^2/J_{n-1})$.