

Mathematical induction as a higher-order logical principle based on permutations of $F \succ F = T$.

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From: en.wikipedia.org/wiki/Higher-order_logic

"First-order logic quantifies only variables that range over individuals; second-order logic, in addition, also quantifies over sets; third-order logic also quantifies over sets of sets, and so on.

For example, the second-order sentence

$$\forall P ((0 \in P \wedge \forall i (i \in P \rightarrow i + 1 \in P)) \rightarrow \forall n (n \in P)) \quad (1.1)$$

expresses the principle of mathematical induction. Higher-order logic is the union of first-, second-, third-, ... , n th-order logic; i.e., higher-order logic admits quantification over sets that are nested arbitrarily deeply."

Remark: The element n th-order logic implies it is a permutation.

We evaluate higher-order logic based on the principle of mathematical induction.

We assume the Meth8/VL4 apparatus and method.

LET: p q r P i n; # necessity, all, \forall ; % possibility, one or some; + Or; - Not Or;
 & And; > Imply, \rightarrow ; < Not Imply, less than, \in ; 1 (%p>#p); 0 (p@p) .
 The designated proof value is T; F contradiction; c falsity; N truth.
 The 16-valued truth tables are row-major and presented horizontally.

Eq. 1.1 is a higher-order logic expression where the entire formula is universally quantified on one set (P) over universally quantified variables (i, n).

Meth8/VL4 treats sets and variables as variables. Therefore Eq. 1.1 can be rendered by inserting quantifiers to modify each occurrence of a variable:

$$(((p@p)<#p)&((\#q<#p)>((\#q+(\%p>#p))<#p)))>(\#r<#p)) ; \quad \begin{matrix} TTTT & TTTT & TTTT & TTTT \end{matrix} \quad (1.2)$$

We examine the antecedent and consequent of Eq. 1.2.

$$((p@p)<#p)&((\#q<#p)>((\#q+(\%p>#p))<#p)) ; \quad \begin{matrix} FFFF & FFFF & FFFF & FFFF \end{matrix} \quad (1.3)$$

$$\#r<#p ; \quad \begin{matrix} FFFF & TCTC & FFFF & TCTC \end{matrix} \quad (1.4)$$

The principle of induction in Eq. 1.2 is tautologous as a permutation by way of the generic format of $F \succ F = T$.