The tacit assumption of continuity of spacetime in quantum gravity

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General relativity and quantum mechanics both have been confirmed by experiments. In order to unify them, a new theory of quantum gravity is searched, but without success.

The current theories of quantum gravity are based on the tacit assumption of a continuous, differentiable Lorentzian spacetime manifold. Little attention has been paid to this assumption, but no theory is complete without the prior verification of the soundness of its underlying assumptions.

Surprisingly, we will see in the following that, whatever is the approach we choose, nothing is corroborating the assumption of a continuous spacetime manifold.

0. Introduction

It seems to be an unnecessary question. It is obvious and may be proved that the space manifold is a continuous threedimensional manifold, and that Newton's space and time were forming one continuous manifold - why should there be any doubt with respect to Lorentzian spacetimes?

Surprisingly, we will see in the following that, whatever is the approach we choose, nothing is corroborating the assumption of a continuous spacetime manifold.

1. The continuity of spacetime is an assumption

This assumption was expressed by Minkowski in his lecture "Space and Time" in 1908:

> *"In order to leave nowhere a gaping void, we imagine ourselves that something perceptible is existent at all places and at every moment."* **[1]**

The assumption of a continuous spacetime manifold seemed so obvious and so natural that since then, no doubts arose with respect to this assumption.

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2. The two postulates of special relativity

In particular, the two postulates of special relativity are not complying with a continuous spacetime manifold:

1. *The laws of physics are the same in all inertial reference frames.*
2. *Speed of light is measured with the same value c in all inertial reference frames.*

Manifestly, both postulates are talking about inertial reference frames (and also about lightlike phenomena), but not about the vacuum between worldlines. These inertial reference frames may be considered as particle worldlines, and an observer may assign coordinates to these worldlines, but the contents of the two postulates is limited to these worldlines, the vacuum is in no way concerned.

Vacuum is described by quantum physics, it is neither defined by special relativity, nor by the curved spacetime theories of general relativity. Example: the Schwarzschild metric introduces the warping of the worldlines by gravity, but vacuum between worldlines is not described.

That means that general relativity itself is contradicting the assumption of a continuous spacetime manifold which would include vacuum points.

3. Vacuum points have no time evolution

What exactly is happening with vacuum points in special relativity?
For this question we consider a Minkowski diagram with two lines of simultaneity.

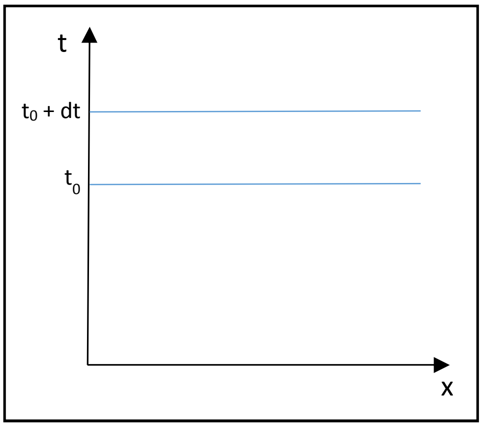

Fig. 1: Two lines of simultaneity seem to be continuous

The two lines of simultaneity seem to be perfectly continuous.

Now we introduce two particle worldlines. The worldline of each particle is determined by its position and its velocity.

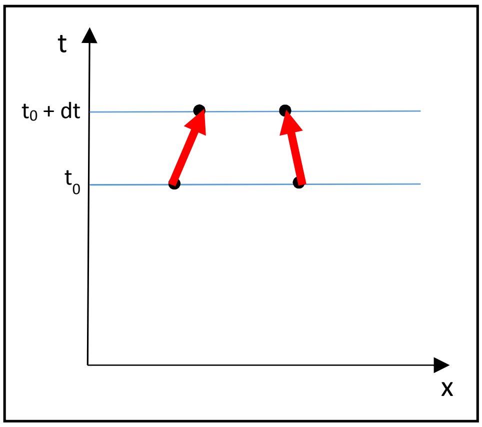

Fig. 2: Two particle worldlines

But what about a vacuum point between both particles? A vacuum point has no defined velocity. It does not travel simply upwards through time, because this would imply a preferred observer. The result: There is no point on the upper line $t_0 + dt$ which corresponds to the vacuum point on the lower line t_0. Vacuum has no time evolution.

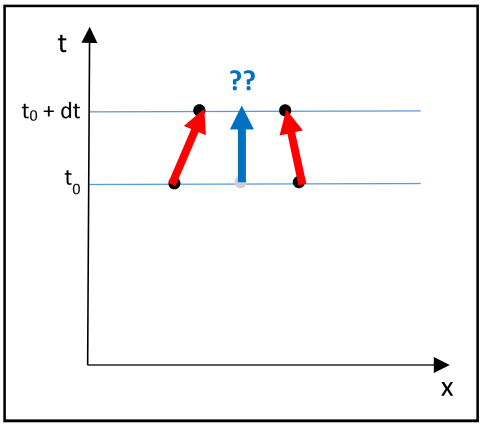

Fig. 3: No point on the upper line corresponds to the vacuum point on the lower line

By consequence, the horizontal simultaneity lines do not include the vacuum between particles, and, in spite of all appearance, they are not continuous.

For lightlike phenomena which are propagating with velocity c (such as fields) the problem is a different one: One could presume that lightlike phenomena are continuous and everywhere, even in the vacuum between particles. But the problem is here that many lightlike phenomena go through the same point such that there is no unambiguous defined point on the upper line which corresponds to the lower line.

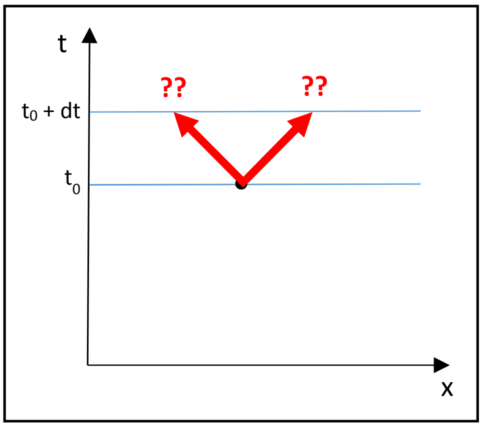

Fig. 4: No unambiguous solution for lightlike phenomena

How is it possible that continuity of Lorentzian spacetime was assumed during 110 years?

The reason is that the continuity of spacetime seems so obvious that human intuition refuses even to imagine the timelessness of the vacuum between worldlines and the absence of a fourdimensional spacetime manifold.

4. Spacetime manifolds are mere observation only

We saw that simultaneity lines seem to be continuous although vacuum is not defined in spacetime. That means that observation is not representing the reality.

In daily life, observation is not identical with reality. Different observers of an object get different views (projections), but their observation is not identical with the reality of the object (such like its overall form, its interior etc.), and even the sum of all possible observations may not show the whole reality.

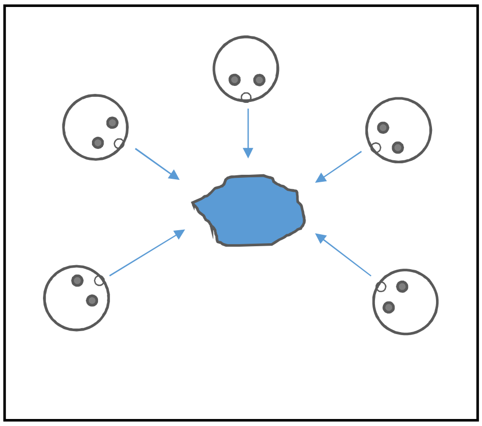

Fig. 5: Five observers in daily life

In the same way, spacetime manifolds are only projections of the real world.

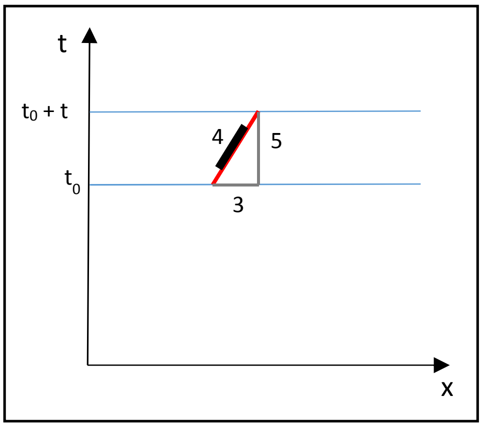

Fig. 6: The observer's view of a particle worldline

Example:

This Minkowski diagram of an observer may show a time lapse of 5 time units for the movement of a particle where other observers (moving near light speed) may observe 6, 7, 8 or even more time units, and an observer following the observed particle will observe only 4 time units.

This shows that Minkowski diagrams are relative and observer-dependent, they are only a projection of reality, and they are hiding the reality of the underlying proper time of the observed particle, which is an absolute value because all observers are agreeing on the underlying proper time.

It is important to notice that even the sum of all imaginable spacetime diagrams does not show such absolute values of proper time which may be considered as reality. One could argue that there is always at least one spacetime diagram showing the proper time of a given particle, i.e. the spacetime diagram of an observer who is sharing the reference frame of the particle. Two reasons are opposed to such an argumentation:

a) We may consider the proper time of more than one different particles. In this case, the spacetime diagram of an observer could not show the proper time of particles which are belonging to different reference frames. Moreover, the topology of several reference frames of several particles cannot be reflected because the spacetime diagram of the observer may be a continuous R^4 manifold whereas there is no continuity between the proper times of several particles, because it is impossible to represent the proper time of several particles in one unique spacetime diagram.

b) We also may consider a lightlike phenomenon whose proper time is zero **[2-4]**: the proper time of lightlike phenomena cannot be observed, all observers will agree on a velocity c of lightlike phenomena, and accordingly they will assign a coordinate time which is bigger than zero. No observer will observe the zero proper time of lightlike phenomena because lightlike phenomena have no reference frame which could be adopted by an observer.

   In the same way, the event horizon and the interior of a black hole may not be observed by observers.

By consequence, even the sum of all imaginable spacetime diagrams is observation only and does not represent the reality. The same does apply also to tensors: Tensors may be transformed from one reference frame to any other, but the underlying reality is not represented by them. For this purpose, we must refer to the absolute values of the spacetime interval and the proper time.

The absolute values of spacetime intervals cannot be represented in spacetime, but they can be represented in space, in the form of worldlines of particles and of lightlike phenomena which are parameterized by their respective proper time, that implies that there is no common time axis.

5. Spacelike intervals would imply square roots of negative numbers

Concerning the spacetime intervals, there are four contradicting concepts. In the diagram below, the spacetime interval could be 4, 4i, 16 or -16.

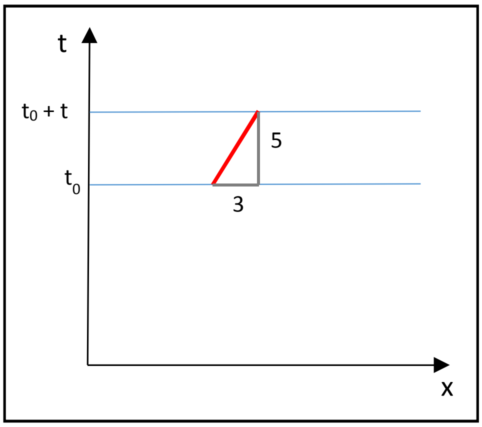

Fig. 7: Four contradicting concepts for the spacetime interval

Since the very beginning of special relativity, there have been applied 4 different ways of calculation for one of the most important institutions of spacetime which is the spacetime interval. The reason is a strange result appearing after extraction of the root of the spacetime interval: Whatever is the signature we choose (+,-,-,- or -,+,+,+), we get imaginary distances in one sense, either for timelike or for spacelike intervals.

Does this strange result mean that spacelike intervals are imaginary?

Misner/ Thorne/ Wheeler **[5]**, instead of discussing this possibility, maintained the real character of spacelike distances by adopting the signature (-,+,+,+), which is considered to be particularly well adapted in cosmology, but there is an increasing number of voices, notably in particle physics, who are adopting the opposed signature (+,-,-,-).**[6]**

Following this tendency (which according to Penrose **[7]** is "more physical"), and by choosing the spacetime intervals to be equal to proper time (i.e. the signature "+,-,-,-" and extraction of the root), we get a very natural result:

There is no continuous spacetime manifold similar to the threedimensional space manifold, but there is only defined a network of worldlines, at the exclusion of vacuum, and all spacetime intervals are object-related. They are not just a distance which would include vacuum distances, but they represent always some proper time of particles traveling through this interval.

Example: For the left interval, we get the proper time "4". For the right interval, we get an imaginary result because a speed beyond speed of light is required.

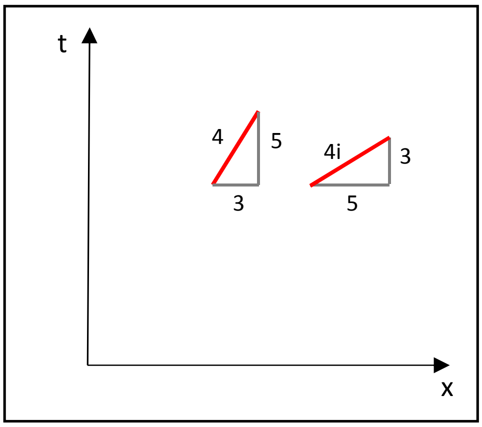

Fig. 8: Two examples of spacetime intervals of particle worldlines

6. No compliance with quantum gravity

Apparently, a continuous (curved) spacetime would not be compatible with quantum gravity, as the quantization of spacetime has not succeeded yet.

7. No "nice" topology

The research for appropriate topologies does not provide significant corroboration of the assumption of continuous spacetime. Quite the contrary, it has been noticed that there is no "nice" topology for Lorentzian spacetimes.**[8][9]** One possible topology is among others the 1+3 topology which is treating space and time separately, corroborating rather the assumption of a threedimensional space manifold than the one of a fourdimensional spacetime manifold. **[10-12]**

8. Absence of experimental corroboration of continuous spacetime

As shown above, the assumption of continuous spacetime was first enunciated by Minkowski in 1908, without reference to experimental evidence, and since then it seems that there has been no experimental support. This seems comprehensible because any experimental evidence in favor of the continuous spacetime would have also to comply with quantum mechanics.

9. Spacetime without continuous manifold

After its refutation, what will take the place of the continuous spacetime manifold?

The answer is twofold:

1. The spacetime manifold must be replaced by an R^3 space manifold.

2. But spacetime still exists, not in the form of a manifold but in the form of a sort of synchronizing <u>operation</u> between worldlines, at the exclusion of vacuum.
   Example: Two particles are approaching the same point in space. If they are reaching the point at the same time, there will be a particle event (a collision), if they go through this point one after the other, there will be no particle event.
   However, if we are basing only on their respective worldline which is parameterized by their respective proper time, it is not possible for us to tell if there will be a particle event or not. Instead, for this purpose it is necessary to insert both worldlines into the spacetime diagram of any arbitrary observer, where they receive a common coordinate time parameter. Such a coordinate time parameter synchronizes both worldlines by taking into account the respective time dilation of both particles. And exactly this is the function of the operation of spacetime.

10. Quantum gravity

The refutation of the continuous spacetime opens a new way to quantum gravity. For this purpose, two items seem to be particular important:

First, the fundamental role of proper time, in particular the zero proper time of lightlike phenomena.

Second, the possibility of the representation of gravity not only in curved spacetime, but also in uncurved spacetime, in the form of gravitational time dilation.

11. References

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