

Refutation of confluence in rewrite systems

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Abstract: We evaluate confluence in two definitions and one theorem, none which is tautologous. This refutes the approach of confluence as a central property of rewrite systems.

We assume the method and apparatus of Meth8/VL4 with Tautology as the designated *proof* value, **F** as contradiction, **N** as truthity (non-contingency), and **C** as falsity (contingency). For results, the 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. Reproducible transcripts for results are available. (See ersatz-systems.com.)

LET p, q, r, s: a, x, y, z;
~ Not, ¬; + Or, ∨, ∪; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ↗, >, ⊃, ⊇, ≧, < Not Imply, less than, ∈, <, <;
= Equivalent, ≡, :=, ⇔, ↔, ≜ @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ∅, M; # necessity, for every or all, ∀, □, L;
(z=z) **T** as tautology, **T**, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;
(%z<#z) **C** as contingency, Δ, ordinal 1; (%z>#z) **N** as non-contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).

Remark 0: For clarity, we distribute the quantifiers to each instance of a variable.

From: Endrullis, J.; Klop, J.W.; Overbeek, R. (2019).
Decreasing diagrams for confluence and commutation.
arxiv.org/pdf/1901.10773.pdf j.endrullis@vu.nl, j.w.klop@vu.nl, roy.overbeek@cwi.nl

A binary relation \rightarrow is called confluent if two cointial reductions (i.e., reductions having the same starting term) can always be extended to cofinal [having same term] reductions, that is:

$$\forall abc.(b \leftarrow a \rightarrow c \Rightarrow \exists d.b \rightarrow d \leftarrow c) \tag{1.1}$$

$$((\#q < \#p) > \#r) > (\#q > (\%s < \#r)); \quad \mathbb{T} \tag{1.2}$$

Definition 5 (Strong confluence). [≡ means empty set, ignored here]

$$\forall axy.\exists z.(a \rightarrow x \wedge a \rightarrow y) \Rightarrow (x \rightarrow \equiv z \leftarrow y) \tag{5.1}$$

$$((\#p > \#q) \& (\#p > \#q)) > (q > (\%s < r)); \quad \mathbb{T} \tag{5.2}$$

Theorem 23:

Proof: ... Finally, the following formula requires all elements, except for [a], to be deterministic:

$$\dots \forall xyz.(a \rightarrow x \wedge a \rightarrow y \wedge a \rightarrow z) \Rightarrow y = z \quad (23.1)$$

$$((q @ (p \& q)) > (q > r)) > (r = s) ;$$

$$\text{TTTT } \mathbf{FFFF} \mathbf{FFTF} \text{TTTT} \quad (23.2)$$

Eqs. 1.2, 5.2, and 23.2 for two definitions and one theorem are *not* tautologous. We evaluate confluence in two definitions and one theorem, none which is tautologous. This refutes the approach of confluence as a central property of rewrite systems.