Refutation of confluence in rewrite systems

Abstract: We evaluate confluence in two definitions and one theorem, none which is tautologous. This refutes the approach of confluence as a central property of rewrite systems.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). For results, the 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. Reproducible transcripts for results are available. (See ersatz-systems.com.)

LET p, q, r, s: a, x, y, z;
~ Not, ¬ ; + Or, ∨, ∪; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ↞;
= Equivalent, ≡, :=, ⟺, ↔, ≜; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;
(%z<#z) C as contingency, Δ, ordinal 1; (%z>#z) N as non-contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).

Remark 0: For clarity, we distribute the quantifiers to each instance of a variable.

From: Endrullis, J.; Klop, J.W.; Overbeek, R. (2019).
Decreasing diagrams for confluence and commutation.
arxiv.org/pdf/1901.10773.pdf j.endrullis@vu.nl, j.w.klop@vu.nl, roy.overbeek@cwi.nl

A binary relation → is called confluent if two coinitial reductions (i.e., reductions having the same starting term) can always be extended to cofinal [having same term] reductions, that is:

$$\forall abc.(b \twoheadleftarrow a \twoheadrightarrow c \Rightarrow \exists d.b \twoheadrightarrow d \twoheadleftarrow c) \quad (1.1)$$

((#q<#p)>#r)>(#q>(%s<#r)) ; `TTTC TTCC TTTC TTCC` (1.2)

Definition 5 (Strong confluence). [$^{=}$ means empty set, ignored here]

$$\forall axy.\exists z.(a \to x \wedge a \to y) \Rightarrow (x \to^{=} z \twoheadleftarrow y) \quad (5.1)$$

((#p>#q)&(#p>#q))>(q>(%s<r)) ; TTCC TT**FF** TTCC TT**FF** (5.2)

Theorem 23:
Proof: ... Finally, the following formula requires all elements, except for [a], to be deterministic:

$$\ldots\ \forall xyz.(a\to x\wedge a\to y\wedge a\to z)\Rightarrow y=z \qquad (23.1)$$

((q@(p&q))>(q>r))>(r=s) ;

TTTT **FFFF** **FF**T**F** TTTT (23.2)

Eqs. 1.2, 5.2, and 23.2 for two definitions and one theorem are *not* tautologous. We evaluate confluence in two definitions and one theorem, none which is tautologous. The refutes the approach of confluence as a central property of rewrite systems.