

Refutation of the parameter free ZFC⁰

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Abstract: We evaluate the parameter free ZFC⁰. We test the parameter free schema of comprehension and of replacement. Neither are tautologous. This refutes ZFC⁰.

We assume the method and apparatus of Meth8/VL4 with Tautology as the designated proof value, **F** as contradiction, **N** as truthity (non-contingency), and **C** as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET $p, r, s, u, v, x, y: \varphi, a, b, x', y', x, y;$
 \sim Not, \neg ; $+$ Or, \vee, \cup ; $-$ Not Or; $\&$ And, \wedge, \cap ; \setminus Not And;
 $>$ Imply, greater than, $\rightarrow, \mapsto, \succ, \supset, \vdash, \models, \twoheadrightarrow$; $<$ Not Imply, less than, $\in, \prec, \subset, \neq, \neq, \leftarrow$;
 $=$ Equivalent, $\equiv, :=, \iff, \leftrightarrow, \triangleq, @$ Not Equivalent, \neq ;
 $\%$ possibility, for one or some, \exists, \diamond, M ; $\#$ necessity, for every or all, \forall, \square, L ;
 $(z=z)$ **T** as tautology, \top , ordinal 3; $(z@z)$ **F** as contradiction, $\emptyset, \text{Null}, \perp$, zero;
 $(\%z<\#z)$ **C** as contingency, Δ , ordinal 1; $(\%z>\#z)$ **N** as non-contingency, ∇ , ordinal 2;
 $\sim(y < x)$ $(x \leq y)$, $(x \subseteq y)$; $(A=B)$ $(A \sim B)$.

From: Hester, J. (2019). Automated ZFC theorem proving with E.
arxiv.org/pdf/1902.00818.pdf hesterj@ufl.edu

ZFC⁰, or parameter free ZFC, is an alternative axiomatization of ZFC where the schemas of comprehension and replacement have been replaced by their parameter free counterparts, and the rest of the axioms remain the same. ZFC⁰ is equivalent to ZFC as every instance of the full axioms of comprehension and replacement can be derived in a finite number of steps in ZFC⁰.

Parameter Free Schema of Comprehension:

Let $\varphi(x)$ be any formula in the language of ZFC with a single free variable x , and let y be some variable not in φ . Then $\forall a \exists y \forall x (x \in y \leftrightarrow x \in a \wedge \varphi(x))$ (1.1)

$$((\#x < \%y) = (x < \#r)) \& (\#r \& (p \& \#x)) ;$$

$$\mathbf{FFFF\ FFFF\ FFFF\ FFFF\ (48),\ FFFF\ FNFN\ FFFF\ FNFN\ (16)} \quad (1.2)$$

Parameter Free Schema of Replacement:

For every formula $\varphi(x, y)$ of the language of ZFC,
 $\forall x \exists y \forall y' (\varphi(x, y') \leftrightarrow y' = y) \rightarrow \forall a \exists b \forall y (y \in b \leftrightarrow \exists x \in a \varphi(x, y))$ (2.1)

$$(((p \& (\#x \& \#v)) = (\#v = \%y)) >$$

$$(\#y < \%s) = ((\%x < \#r) \& (p \& (\%x \& \#y)))) > \#(p \& (x \& y)) ;$$

$$\mathbf{FFFF\ FFFF\ FFFF\ FFFF\ (32),}$$

$$\mathbf{NNNN\ NNNN\ FFFF\ FFFF\ (4),\ FFFF\ FFFF\ FFFF\ FFFF\ (4),}$$

$$\mathbf{NNNN\ NNNN\ FFFF\ FFFF\ (4),\ FFFF\ FFFF\ FFFF\ FFFF\ (4),}$$

$$\mathbf{NNNN\ NNNN\ FNFN\ FNFN\ (4),\ FNFN\ FNFN\ FNFN\ FNFN\ (4),}$$

$$\mathbf{NNNN\ NNNN\ FNFN\ FNFN\ (4),\ FNFN\ FNFN\ FNFN\ FNFN\ (4)} \quad (2.2)$$

Eqs. 1.2 and 2.2 as rendered are not tautologous. This refutes ZFC⁰.