Can the idea of a complex-valued potential explain the *Zitterbewegung*?

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7 February 2019

Preliminary note: Because this paper is so short, I have no table of contents – but it is structured in four easy parts. The introduction below also serves as a summary.

Introduction and summary : The *Zitterbewegung* model of an electron is intuitive and attractive because – when combining the idea of a motion in two dimensions (the *Zitter*) with the idea of a pointlike charge – we get all of the previously unexplained values of the observables (the Compton radius, its angular momentum and magnetic moment, and the correct value for the gyromagnetic ratio) out of a limited set of very fundamental equations – the force law and Einstein's E = mc^2 formula, basically. However, the core problem of the model is this very obvious question: what keeps the pointlike charge in its circular orbit? We can develop metaphors but – when everything is said and done – we do not have springs or pistons or any other mechanical devices in space.

This paper explores a simple idea: if we can describe the particle itself by Euler's wavefunction – exploring different *aspects* of its reality, such as the position of the pointlike charge, and the nature of the force that makes it move along its circular orbit – then we should, perhaps, also explore how we can use it to describe the nature of the *space* that comes with the particle.

The basic idea is this: when we describe *physical* space (as opposed to a purely mathematical space – coordinate space, that is), we will usually describe its *nature* in terms of some *potential*, whose derivative will then give us the force acting on our particle. Hence, if the force on our particle can be described by a complex function, perhaps we should try to describe potential in terms of a complex-valued function as well. So that is what we will do here.

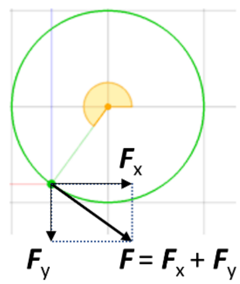
I. The *Zitterbewegung* model

comes in various versions. Ours is the simplest of models – it basically reflects Dirac's own description of it, which we may usefully quote before illustrating it. To be precise, it was Erwin Schrödinger who stumbled upon it when he was exploring solutions to Dirac's wave equation for free electrons. As it earned him a share in the 1933 Nobel Prize which Paul Dirac then received for "the discovery of new productive forms of atomic theory", we will just quote Dirac's summary of Schrödinger's discovery:

> "The variables give rise to some rather unexpected phenomena concerning the motion of the electron. These have been fully worked out by Schrödinger. It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small. But one must believe in this consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by

experiment." (Paul A.M. Dirac, *Theory of Electrons and Positrons*, Nobel Lecture, December 12, 1933)

We think the illustration below captures this basic idea. Think of the green dot as representing the circular oscillatory motion of the pointlike charge. The combined idea of the pointlike charge and its motion constitutes the idea of an electron (in free space, obviously).[1] The model basically combines the concept of a pointlike charge and Wheeler's idea of mass without mass. Hence, the mass of the electron is the equivalent mass of the energy in the oscillation of the pointlike charge.



The pointlike charge is, obviously, driven by a force $\boldsymbol{F}$ – which must be electromagnetic, because the force has only a charge to grab onto. We think of this charge as a pointlike object that has no rest mass. Hence, the charge spins around at the speed of light. We have a dual view of the reality of the wavefunction here.

1. On the one hand, it will describe the physical position (i.e. the *x*- and *y*-coordinates) of the pointlike charge – the green dot in the illustration, whose motion is described by:

$$\boldsymbol{r} = a\cdot e^{i\theta} = x + i\cdot y = a\cdot\cos(\omega t) + i\cdot a\cdot\sin(\omega t) = (x, y)$$

As such, the (elementary) wavefunction may be viewed as an implicit function: it is equivalent to the $x^2 + y^2 = a^2$ equation, which describes the same circle.

2. On the other hand, the *zbw* model implies the circular motion of the pointlike charge is driven by a tangential force, which we write as:

$$\boldsymbol{F} = F_x\cdot\cos(\omega t+\pi/2) + i\cdot F_x\cdot\sin(\omega t+\pi/2) = F\cdot e^{i(\theta+\pi/2)}$$

The line of action of the force is the orbit because a force needs something to grab onto, and the only thing it can grab onto in this model is the oscillating (or rotating) charge. We think of $\boldsymbol{F}$ as a composite force: the resultant force of two perpendicular oscillations.

II. A metaphor for such oscillation is the idea of two springs in a 90-degree angle working in tandem to drive a crankshaft. The 90-degree angle ensures the independence of both motions. The kinetic and potential energy of *one* harmonic oscillator add up to $E = m\cdot a^2\cdot\omega^2/2$. If we have two, we can drop the ½ factor. We can then boldly equate the $E = mc^2$ and $E = m\cdot a^2\cdot\omega^2$ formulas to get the *zbw* radius.[2] We can

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[1] We have extended the model to electron orbitals in previous papers. We have also developed a new photon model re-using some of the concepts – most notably the idea of the integrity of a cycle.

[2] The reader will immediately wonder if the $E = m\cdot a^2\cdot\omega^2/2$ is relativistically correct and, if not, what it implies. We explored this objection in one of our very first papers (*The Wavefunction as an Energy Propagation Mechanism*, http://vixra.org/abs/1806.0106) and, hence, will not repeat ourselves here.

think of this as follows. The *zbw* model – which is derived from Dirac's wave equation for free electrons – tells us the velocity of the pointlike charge is equal to c. If the *zbw* frequency is given by Planck's energy-frequency relation ($\omega = E/\hbar$), then we can combine Einstein's $E = mc^2$ formula with the radial velocity formula ($c = a \cdot \omega$) and, hence, we get the *zbw* radius, which is nothing but the (reduced) Compton wavelength – or the Compton *radius* of the electron:

$$a = \frac{\hbar}{\mathrm{m}c} = \frac{\lambda_C}{2\pi} \approx 0.386 \times 10^{-12}\ \mathrm{m}$$

The amount of *physical* action – which we will denote by S as per the usual convention – that is associated with one loop along the *zbw* circumference over its cycle time is equal to Planck's constant:

$$S = F \cdot \lambda_C \cdot \mathrm{T} = \frac{\mathrm{E}}{\lambda_C} \cdot \lambda_C \cdot \frac{1}{f_C} = \mathrm{E} \cdot \frac{h}{\mathrm{E}} = h$$

Planck's constant h is equal to $6.62607015 \times 10^{-34}$ J·s. Hence, it is a small unit - but small and large are relative. In fact, because of the tiny time and distance scale, we have a rather enormous force here. We can, effectively calculate the force because the energy in the oscillator must be equal to the magnitude of the force times the length of the loop, we can calculate the magnitude of the force, which is – effectively – rather enormous in light of the sub-atomic scale:

$$\mathrm{E} = \mathrm{F}\lambda_C \Leftrightarrow F = \frac{\mathrm{E}}{\lambda_C} \approx \frac{8.187 \times 10^{-14}\ \mathrm{J}}{2.246\ \times 10^{-12}\ \mathrm{m}} \approx 3.3743 \times 10^{-2}\ \mathrm{N}$$

The associated current is equally humongous:

$$\mathrm{I} = \mathrm{q_e}f = \mathrm{q_e}\frac{\mathrm{E}}{h} \approx (1.6 \times 10^{-19}\ \mathrm{C})\frac{8.187 \times 10^{-14}\ \mathrm{J}}{6.626 \times 10^{-34}\ \mathrm{Js}} \approx 1.98\ \mathrm{A}\ (ampere)$$

A household-level current at the sub-atomic scale? The result is consistent with the calculation of the magnetic moment, which is equal to the current times the area of the loop and which is, therefore, equal to:

$$\mu = \mathrm{I} \cdot \pi a^2 = \mathrm{q_e}\frac{\mathrm{m}c^2}{h} \cdot \pi a^2 = \mathrm{q_e}c\frac{\pi a^2}{2\pi a} = \frac{\mathrm{q_e}c}{2}\frac{\hbar}{\mathrm{m}c} = \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar$$

It is also consistent with the presumed angular momentum of an electron, which is that of a spin-1/2 particle. As the oscillator model implies the effective mass of the electron will be spread over the circular disk, we should use the 1/2 form factor for the moment of inertia (I). We write:

$$\mathrm{L} = I \cdot \omega = \frac{ma^2}{2}\frac{c}{a} = \frac{mc}{2}\frac{\hbar}{mc} = \frac{\hbar}{2}$$

We now get the correct g-factor for the pure spin moment of an electron:

$$\boldsymbol{\mu} = -\mathrm{g}\left(\frac{\mathrm{q_e}}{2\mathrm{m}}\right)\mathbf{L} \Leftrightarrow \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar = \mathrm{g}\frac{\mathrm{q_e}}{2\mathrm{m}}\frac{\hbar}{2} \Leftrightarrow \mathrm{g} = 2$$

We have analyzed and generalized these results elsewhere[3] and, hence, we will refer to our previous papers. We only wanted to recap the fundamental results of the *Zitterbewegung* model – so as to show how attractive it is. However, as mentioned in our introduction, the model does trigger a very fundamental question – which we need to address head-on.

III. What is the nature of the force?

Where does it come from? We know it must be electromagnetic in its nature – because it grabs onto a force – but where does it come from?

In classical mechanics, a force may be defined as the (negative of the) derivative of a potential. Such potential may be gravitational or electrostatic. We write:

$$-\mathrm{d}U/\mathrm{d}x = F(x) = F_x$$

If we're considering the y-direction, then we write $-\mathrm{d}U/\mathrm{d}y = F(y) = F_y$.

What if we would – somehow – think of the $a \cdot e^{i\theta}$ function as some *complex* potential. Let us forget about the coefficient *a* for a while (we can plug it back in at a later stage), so we write:

$$\mathbf{U} = e^{i\theta}$$

Let us take the derivative in regard to the variable here, which is… What? It is the angle θ. It is a *real* number, so we will *not* be calculating the usual derivative of a complex exponential, which is $\mathrm{d}(e^z)/\mathrm{d}z = e^z$, with *z* a complex number. Instead, we calculate:

$$-\mathrm{d}\mathbf{U}/\mathrm{d}\theta = -\mathrm{d}(e^{i\theta})/\mathrm{d}\theta = -\mathrm{d}(\cos\theta + i\cdot\sin\theta)/\mathrm{d}\theta = -\mathrm{d}(\cos\theta)/\mathrm{d}\theta - i\cdot\mathrm{d}(\sin\theta)/\mathrm{d}\theta$$

$$= \sin\theta - i\cdot\cos\theta = \cos(\theta-\pi/2) + i\cdot\sin(\theta-\pi/2)$$

We get the sine and cosine factors of our formula, except the sign is right: the phase factor should be $+\pi/2$ instead of $-\pi/2$. That problem is solved if we drop the minus sign in front of the $-\mathrm{d}\mathbf{U}/\mathrm{d}\theta$ derivative:

$$\mathrm{d}\mathbf{U}/\mathrm{d}\theta = \mathrm{d}(e^{i\theta})/\mathrm{d}\theta = \mathrm{d}(\cos\theta + i\cdot\sin\theta)/\mathrm{d}\theta = \mathrm{d}(\cos\theta)/\mathrm{d}\theta + i\cdot\mathrm{d}(\sin\theta)/\mathrm{d}\theta$$

$$= -\sin\theta + i\cdot\cos\theta = \cos(\theta+\pi/2) + i\cdot\sin(\theta+\pi/2)$$

Why would we drop the minus sign? One may think it could be related to the other mathematical possibility: the rotation may be clockwise rather than counterclockwise. The mathematical formalism works out equally well, but it does not explain why we should drop the minus sign in front of the derivative. However, *if* we acknowledge there would be a minus sign if we would have adopted the convention of measuring angles clockwise rather than counterclockwise, *then* we see it's just a matter of convention, effectively.

IV. Why is this important?

If we take Euler's function to actually represent an elementary charged particle (practically speaking, we are talking an electron here), then the *Zitterbewegung* model suggests it comes with its own space. It must – if we want to address the key concern in regard to the

[3] See: *The Emperor Has No Clothes: A Classical Interpretation of Quantum Mechanics*, http://vixra.org/abs/1901.0105.

model, which is nicely summarized in one of Dr. Burinskii's very first communications to me. He effectively wrote the following to me when I first contacted him on the viability on the model:

> "I know many people who considered the electron as a toroidal photon[4] and do it up to now. I also started from this model about 1969 and published an article in JETP in 1974 on it: "Microgeons with spin". Editor E. Lifschitz prohibited me then to write there about *Zitterbewegung* [because of ideological reasons[5]], but there is a remnant on this notion. There was also this key problem: what keeps [the pointlike charge] in its circular orbit?"[6]

He noted that this fundamental flaw was (and still is) the main reason why had abandoned the simple *Zitterbewegung* model in favor of the much more sophisticated Kerr-Newman approaches to the (possible) geometry of an electron. I am reluctant to make the move he made – mainly because I prefer simple math to the rather daunting math involved in Kerr-Newman geometries – and so that is why I am continuing to explore alternative explanations – such as this one.

Does it make sense to try? I am not sure. The idea of a complex-valued potential may or may not provide the ultimate answer – but it sure does cater to the idea of a particle coming with its own space. The *nature* of this space is – quite simply – this new concept: a complex-valued potential. The first reaction of the reader is predictable: this must be nonsense. I invite the reader to think about *why* he would say that – because my own initial reaction to my thoughts was the same: this is ridiculous. However, I then realized that my instinctive objection to my own thoughts was that it is somewhat hard to distinguish ontological or mathematical concepts here from what might (or might not) be reality – or *physical* concepts, I should say. In fact, the ambiguity is in the concept of a potential itself. It is *less* tangible than a force. It is like thinking of a force without thinking simultaneously about what it's going to grab onto – which is not an easy exercise. To put it differently, I have a strong feeling that my train of thought here involves implicit tautologies. Having said that, tautologies – when made explicit – may bring new insights.

After my initial skepticism, I also started to think that the idea might make some sense because of the following. The *zbw* model offers a basic *interpretation* of the wavefunction by noting the various *aspects* of the (possible) reality that might correspond to the wavefunction – which we referred to as the *dual view* of a wavefunction. That dual view consisted of (1) a description of the *position* of our pointlike particle and (2) a description of the *force* that makes it move. Both descriptions are descriptions in terms of a complex-valued function: the wavefunction itself. Hence, it would seem to be logical that we develop a *third* view now: the wavefunction as a description of the physical space that comes with the particle.

END

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[4] This is Dr. Burinskii's terminology: it does refer to the *Zitterbewegung* electron: a pointlike charge with no mass in an oscillatory motion – orbiting at the speed of light around some center.

[5] This refers to perceived censorship from the part of Dr. Burinskii. In fact, some of what he wrote me strongly suggests some of his writings have, effectively, been suppressed because – when everything is said and done – they do fundamentally question – directly or indirectly – some key assumptions of the mainstream interpretation of quantum mechanics.

[6] Email from Dr. Burinskii to the author dated 22 December 2018.