# Correlation of the Fine-Structure Constant to the Cosmic Horizon and Planck Length

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**Abstract**

The following paper derives the fine-structure constant. This derivation suggests that the fine-structure constant can be theoretically determined as the spectrum range of all the energy modes fitting inside the observable universe. This corresponds to the number of allowed radiation modes of a particle from the cosmic horizon down to Planck length. Additionally, an association between Newton's Law of Gravity and Coulomb's Law suggests there is a connection between mass and charge via the fine-structure constant.

## 1 Introduction

The fine-structure constant (FSC) is subject to multiple physical interpretations and is a recurrent topic for vivid scientific debates [1] [4]. The fundamental nature of the fine-structure constant has remained unclear since its discovery by A. Sommerfeld where he defined this number as the strength of the electromagnetic interaction between elementary charged particles. Furthermore, it has not been clarified whether the fine-structure constant may vary in spacetime [2].

This paper introduces a new perspective suggesting that the nature of the FSC is correlated to information horizon radiation which is emitted from the cosmic particle horizon. Similar to Hawking radiation [5], it can be assumed that radiation emits from an information horizon and this establishes a discrete wave spectrum with allowed wavelengths fitting within the nodes of the horizon confinement [6] [7]. Such radiation could be assumed to cause energy gradients in the realm of virtual particles which may indicate the establishment of forces inside the vacuum. The existence of such radiation is assumed and by wave superposition, a fairly precise FSC will be derived. This paper provides the physical interpretation of the FSC and a more precise FSC which considers the effect of the fundamental strong force. The fundamental FSC can be physically derived utilizing the Lambert $W$-function and a simple ratio of the current size of the cosmic particle horizon and Planck length. Finally, an association between Coulomb's law and Newton's gravity law will be analyzed as well.

# 2 Method

## 2.1 Derivation for fundamental alpha

Using the uncertainty principle, it is suggested that in-between two objects, or boundary conditions, and at a defined distance, virtual particles have energy/momentum waves that are associated to $\Delta x$, and momentum, $\Delta p$. These virtual particles may transfer momentum in a confinement situation within the boundaries. Here, assuming a discrete spectrum with nodes at the horizon [4] [5], a certainty is established where the allowed waves are defined precisely for every wavelength (energy) and this corresponds to the position of the particles.

Now assume there are two elementary probe charges located each at the edge of the observable universe and count all the waves between these two objects. The charges of these particles are inconsequential since they will merely change the force direction and the total waves would be the same in the region. Count all the waves from Planck length, $l_p$, to the cosmic diameter, $\Theta = 8.8 \cdot 10^{26}$ m multiplied by $\pi/2$. This is done using the assumption that the waves are a consequence of oscillations within the horizon which propagate a circular wave from the boundary to the middle of the confinement. The mode with the highest frequency would be the Planck Length while the mode with the greatest wavelength would be the cosmic horizon multiplied by $\pi/2$. This is due to its half circular nature and the momentum direction is a vector from the horizon to the particle location in the middle of the confinement.

$$\Delta x \Delta p = \frac{\hbar}{2} \tag{1}$$

Now solve in terms of the change in momentum.

$$\Delta p = \frac{\hbar}{2\Delta x} \tag{2}$$

Use the energy formula for a photon $\Delta E/c = \Delta p$ and solve in terms of $\Delta E$.

$$\Delta E = \frac{\hbar c}{2\Delta x} \tag{3}$$

Here $\Delta x_{max} = \pi\Theta/2$ since the greatest uncertainty from the averaged probablity of a particle in position of the superimposed wave is located in the middle of the directional span of modes. The horizon, in absence of a directional acceleration, is a circle.

$$\Delta E = \frac{\hbar c}{\pi\Theta} \tag{4}$$

However (4) is simply the maximum distance $\Delta x$ can be. Therefore, using this knowledge, plug in for $\Delta x = kl_p$ in order to count all the waves from Planck length to the circular horizon between the two probe charges up to $N$. The observer is positioned in the middle so energy of the waves will be counted on both sides.

$$\sum_{k=1}^{N} \Delta E_{tot} = \frac{2\hbar c}{2l_p} + \frac{2\hbar c}{4l_p} + \cdots + \frac{2\hbar c}{2Nl_p} \tag{5}$$

Now look at the summation for the total waves, $K$, to count the modes of the superimposed waves of the two charges up to $N$. Where $N$ is the number of waves.

$$K = \sum_{k=1}^{N} \frac{1}{\Delta x} = \frac{1}{l_p} + \frac{1}{2l_p} + \cdots + \frac{1}{Nl_p} \tag{6}$$

Next replace $N$ with $\frac{\Theta\pi}{2l_p}$ to compute all the waves from Planck length to the observable universe.

$$K = \sum_{k=1}^{\Theta\pi/(2l_p)} \frac{1}{k} \tag{7}$$

Use the closed form approximation for a harmonic series formula namely $\sum_{k=1}^{N} \frac{1}{k} \approx \ln(Ne^{\gamma})$ where Euler–Mascheroni's constant is denoted as $\gamma$. Note, for large $\Theta$, this approximation becomes an equality.

$$K = \ln\left(\frac{\Theta\pi e^{\gamma}}{2l_p}\right) \tag{8}$$

This is the total amount of waves for the energy between elementary particles. Insert alpha, the coupling factor, as a power factor inside the equation. Recall, $\alpha$ is defined as the strength of the electromagnetic force between elementary particles.

$$K_{em} = \ln\left(\frac{\Theta\pi e^{\gamma}\alpha}{2l_p}\right) \tag{9}$$

The maximum wavelength between the two charges will have the following energy formula: $\frac{\hbar c}{2\Theta/2}$. Notice the denominator is simply the distance between the two charges. Recall the particle lies in the middle of the confinement between the two probe charges. Next, divide this by the reciprocal of total waves for the electromagnetic force. Again, $\alpha$ is the strength of the electromagnetic force so the waves will have this power factor inside. Recall the formula for the electric potential for two charges is $U = \frac{k_e q_1 q_2}{r}$. This can be rewritten using $k_e = \frac{\alpha \hbar c}{\mathbf{e}^2}$ and using two elementary charges, $\mathbf{e}$, reduces to the following form $U = \frac{\hbar c \alpha}{r}$.

$$U = \frac{\hbar c}{\Theta} \frac{1}{\ln(\frac{\pi \Theta e^{\gamma} \alpha}{2 l_p})} \tag{10}$$

Alpha can now be identified to the logarithmic ratio in the denominator. Namely,

$$\alpha = \frac{1}{\ln(\frac{\pi \Theta e^{\gamma} \alpha}{2 l_p})} \tag{11}$$

This special type of equation can be solved by utilizing the Lambert $W$-function denoted as $W$.

$$\frac{1}{\alpha} = W\left(\frac{\pi e^{\gamma} \Theta}{2 l_p}\right) \tag{12}$$

What has been found can be denoted as alpha fundamental, $\alpha_{fund}$. It can be correlated with the total fundamental electric energy and is the superposition of all the waves between the two charges where $1/\alpha_{fund} = 138.2609014473$. This is close to the value of the experimental of $1/\alpha_{exp} = 137.035999173$ [8]. Additionally, (11) can also be solved by Newton's method or graphing to yield the same result.

## 2.2 Derivation for Converging Alpha Using Strong Force

Although the energy of the electromagnetic force spans the entire observable universe, one critical factor needs to be included. The strong force negates the electromagnetic force in a short region, so the relevant waves must be subtracted. This region exists from approximately 2.385 fm to 0.7 fm [3]. Since the actual details of the strong force behavior are not fully understood yet, it appears reasonable to use a model similar to the electromagnetic energy wave. Additionally, a small region of weak force exists but it decreases exponentially and has such an extremely short range ($10^{-18}$ fm) that it can be neglected. Subtract the waves in the region of the strong force from 2.385 fm to 0.7 fm. The following property of logarithms, $ln(a/l_p) - ln(b/l_p) = ln(a/b)$, is used to subtract the waves in a region. Notice all other constants cancel out during the division including Planck length, $e^{\gamma}$ and $\pi$. So the formula results into the logarithmic ratio in the region.

$$\alpha = \frac{1}{\ln(\frac{\pi \Theta e^{\gamma} \alpha}{2 l_p}) - \ln(\frac{2.385}{0.7})} \tag{13}$$

Combine the two logarithmic terms using $ln(a) - ln(b) = ln(a/b)$.

$$\alpha = \frac{1}{\ln(\frac{\pi 0.7\Theta e^{\gamma}\alpha}{4.77 l_p})} \tag{14}$$

The result is once again the Lambert function denoted as $W$ such that,

$$\frac{1}{\alpha} = W\left(\frac{0.7\pi e^{\gamma}\Theta}{4.77 l_p}\right) \tag{15}$$

Finally the equation for the theoretical fine-structure constant will result in the following value.

$$\frac{1}{\alpha} = 137.0350273791 \tag{16}$$

This theoretical fine-structure constant has an error of approximately $7 \cdot 10^{-6}$ with respect to the experimental value of $1/\alpha_{exp} = 137.035999139$.

### 2.3 Fine Structure Constant's Relationship to Gravity

Coulomb's Law and Newton's Gravity Law have been often compared for their striking similarity. Here, this comparison is analyzed further to see if there is any connection to the fine-structure constant.

Start by associating the two forces in a ratio. Begin by simplifying both equations $F_C = \frac{k_e q_1 q_2}{r^2}$ and $F_G = \frac{GMm}{r^2}$ by taking both masses and charges to unity. Note $k_e$ and $G$ are Coulomb's constant and the Gravitational constant, respectively.

$$F_C = \frac{k'_e}{r^2} \tag{17}$$

$$F_G = \frac{G'}{r^2} \tag{18}$$

Note that $k'_e$ and $G'$ have units of [N$\cdot$ m]. Take the ratio of the two forces to get the power factor between them.

$$\frac{F_G}{F_C} = \frac{G'}{k'_e} \tag{19}$$

This ratio is the relationship of the two forces with the standard SI units. Next use the same method as in section 2.1 to compute the waves between the two particles. Next, use the stoney mass $m_s$ in order to relate Coulomb' Law and Newton's Gravity Law. It should be noted that both forces are equivalent for elementary charges and stoney masses, respectively. Since this situation currently assumes 1 [kg] of mass, the square of the stoney mass will be divided to find the total number of stoney particles creating the waves. Therefore, it will be the total waves (8), the force power factor, and the stoney mass particles multiplied together as indicated below.

$$\frac{1}{\alpha} = \ln\left(\frac{\pi e^{\gamma}\Theta}{2l_p} \cdot \frac{G'}{k'_e} \cdot \frac{1^2}{m_s^2}\right) \tag{20}$$

Replace the stoney mass $m_s = \sqrt{\alpha}m_p$ where $m_p$ is Planck mass and $k'_e = \alpha\hbar c/\mathbf{e}^2$. Notice the units of $k'_e$ is in [N· m] and the elementary charge $\mathbf{e}$ is unitless.

$$\frac{1}{\alpha} = \ln\left(\frac{\pi e^{\gamma} G'\Theta\mathbf{e}^2}{2\hbar c l_p \alpha^2 m_p^2}\right) \tag{21}$$

Finally replace $G' = \hbar c/m_p^2$ using Planck's gravitational constant relation and simplify. Note $m_p^2$ is unit-less here.

$$\frac{1}{\alpha} = \ln\left(\frac{\pi e^{\gamma}\hbar c\Theta\mathbf{e}^2}{2\hbar c l_p \alpha^2 m_p^4}\right) \tag{22}$$

Simplify to obtain the following formula which highlights the connection between Newton's Gravity and the fine-structure constant using only fundamental constants.

$$\frac{1}{\alpha} = \ln\left(\frac{\pi e^{\gamma}\Theta\mathbf{e}^2}{2l_p \alpha^2 m_p^4}\right) \tag{23}$$

This equation can be solved using a slightly different product log $W$-Lambert function. Here, $\alpha$ can be written as the following.

$$\frac{1}{\alpha} = -2W_{-1}\left(-\frac{\sqrt{2l_p m_p^4}}{2\sqrt{\pi e^{\gamma}\Theta\mathbf{e}^2}}\right) \tag{24}$$

The computed value for $\alpha$ is the following. Additionally, (23) can also be solved by graphing or iteration like Newton's method.

$$\frac{1}{\alpha} = 137.0380609661 \tag{25}$$

# 3 Discussion

The fine-structure constant error in section 2.2 most likely comes from the strong force region. The effective force in this short region is estimated to be around $(2.385 - 0.7)$ fm although it is known it varies greatly. Additionally, the weak force has a range of $10^{-18}$ m and was neglected due to its small effect. Finally, the energy from gravity can also be neglected for sections 2.1 and 2.2 for a similar reason due to its weak strength and its small mass. It seems all the energy comes from the same well and the sum of all the modes of the spectrum must be equivalent to the alpha. In addition, by correlating Newton's Gravity Law and Coulomb's law, the fine-structure constant can be found using the stoney mass. It seems here, the energy well goes towards gravity while the error could be originated from slight deviations in the measurement of the cosmic horizon. Furthermore, getting a matching value of $\alpha_{exp}$ using (23) yields $\Theta = 8.78214 \cdot 10^{26}$ m.

As for a physical interpretation, the equations developed in this paper seem to suggest that a horizon could be acting like a membrane vibrating where the allowed modes are distributed in length over the distance of the horizon's surface. This scenario could be the opposite phenomenon of waves caused by the impact of drop on a flat water surface; a situation where the waves propagate from the outside into the middle rather instead of the point of impact.

# 4 Conclusion

The physical nature of the fine-structure constant seems determined from the energy superposition of all the waves between the objects spanning the entire cosmic horizon. The discovery of the precise behaviors in the short spectrum for both the strong force and weak force should allow full convergence to $\alpha$. In addition, there is a correlation of the fine-structure constant with Newton's Gravity law and Coulomb's law linking mass to charge. The theoretical equations solving for $\alpha$ can be described using Lambert $W$ functions.

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