$\lim_{n\to\infty} - ln(n) + \sum_{k=1}^{n} \frac{1}{k}$ = 0.5772156649...

I know I can make this into two limits:

$\lim_{x\to\infty} - ln(x) + \lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{k}$

I know ln x can be defined as a limit a rearranged version of the one on wikipedia under properties to avoid a 0/0, or undefined.

The this limit is equal to natural log $\lim_{n\to 0} \frac{x^n - 1}{n}$ which can be rearranged $\lim_{n\to 0}(x^n - 1)\frac{1}{n}$ using the distributive property this can be made into $\lim_{n\to 0} \frac{1}{n}x^n - \frac{1}{n}$ finally, this can be put as an infinite limit by inverting every n in the limit $\lim_{n\to\infty} nx^{\frac{1}{n}} - n$

Now I can replace ln(x) to make this double limit

$\lim_{x\to\infty}(\lim_{n\to\infty} nx^{\frac{1}{n}} - n)$

Using simplification because all variable in the function are approaching infinity, I can now put this as one limit

$\lim_{n\to\infty} nn^{\frac{1}{n}} - n$

Breaking this down into several limits using order of operation

$\lim_{n\to\infty} n \lim_{n\to\infty} n^{\frac{1}{n}} - \lim_{n\to\infty} n$

I know $\lim_{n\to\infty} n^{\frac{1}{n}}$ approaches to 1 by using a calculator.

The limit $\lim_{n\to\infty} n$ can be solved by direct substitution just making it ∞ . Wolfram Alpha proof here.

That leaves me with

$\infty \cdot 1 - \infty$

Which is 0

Thus I can now replace the $\lim_{x\to\infty} ln(x)$ to be 0 this leaves me with the sum and limit

$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{k}$

Which can just be put as

$\sum_{k=1}^{\infty} \frac{1}{k}$ = 0.5772156649…

Anywho I should be off stuff like this I'm meh at especially late at night I don't expect to be perfect