Refutation of reverse mathematics on measurability theory and computability theory

Abstract: We evaluate the authors' definition of reverse mathematics in their anticipation of applying it to measurability and computability theory. The argument taking two equations to define reverse mathematics is *not tautologous.* Therefore to apply it to measure and computability theory is meaningless.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞;
= Equivalent, ≡, :=, ⟺, ↔, ≜; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;
(%z<#z) `C` as contingency, Δ, ordinal 1; (%z>#z) `N` as non-contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆y); (A=B) (A~B).

Note: To preserve clarity, we usually distribute quantifiers to each variable so designated.

From: Normann, D.; Sanders, S. (2019).
Representations in measure theory: between a non-computable rock and a hard to prove place.
arxiv.org/pdf/1902.02756.pdf arxiv.org/pdf/1902.02756.pdf

2. Preliminaries
2.1. Reverse Mathematics [higher-order RM in *higher-order* arithmetic]

The aim of RM is to identify the minimal axioms needed to prove theorems of ordinary, i.e. non-set theoretical, mathematics. ...

To formalise this idea [is] the collection of all finite types T, defined by the two clauses:

(i) 0 ∈T and (ii) If σ, τ ∈T then (σ → τ) ∈T, (2.1.1.1)

We write Eq. 2.1.1.1 as: The two clauses of (i) and (ii) imply all finite types.

LET p, q, r, s: n, T, τ, σ.

(((p@p)<q)&(((s&r)<q)>((s>r)<q)))>#q ;
`TTTT TTTT TTTT TTTT` (2.1.1.2)

where 0 is the type of natural numbers, and σ → τ is the type of mappings from objects of type σ to objects of type τ.

In this way,

$1 \equiv 0 \rightarrow 0$ is the type of functions from numbers to numbers, and
where $n + 1 \equiv n \rightarrow 0$. (2.1.2.1)

We write Eq. 2.1.2.1 as: Ordinal one is equivalent to zero implying zero, and where p plus ordinal one is equivalent to p implying zero.

((%p>#p)=((p@p)>(p@p)))&((p+(%p>#p))>(p>(p@p))) ;
NFNF NFNF NFNF NFNF (2.1.2.2)

We state the argument of the text as: Eq. 2.1.1.1 implies Eq. 2.1.2.1. (2.1.3.1)

((((p@p)<q)&(((s&r)<q)>((s>r)<q)))>#q)>
(((%p>#p)=((p@p)>(p@p))) & ((p+(%p>#p))>(p>(p@p)))) ;
NFNF NFNF NFNF NFNF (2.1.3.2)

Remark 2.1: Eq 2.1.3.2 as rendered is *not* tautologous. This means the definition of reverse mathematics is refuted. Therefore to apply it to measure and computability theory is meaningless.