Refutation of the method of forcing

Abstract: We evaluate two examples of paradoxes to be resolved by the method of forcing. The first is *no*t tautologous, and the second is a contradiction so *not* proved as a paradox. This means the forcing method is refuted because it cannot coerce the two examples into abstract proofs and with the ultimate goal to produce larger truth values. What follows is that the forcing method is better suited for paraconsistent logics which are *non* tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞;
= Equivalent, ≡, :=, ⟺, ↔, ≜; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;
(%z<#z) C as contingency, Δ, ordinal 1; (%z>#z) N as non-contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

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1. Introduction: Let us begin with two thought experiments. ... First consider the following "paradox" ... in more formal language we have that, "for all z∈ R, *almost surely* Z≠z", while "*almost surely* there exists a z∈ R, Z=z." (1.1.1)

Remark 1.1.1: We interpret "almost surely" to mean "possibly".

LET p, q, r: z, Z, R

((#p<r)>(q@p))&((%p<r)>(q=p)) ; T**F**NC TTTT T**F**NC TTTT (1.1.2)

Next suppose ... [i]n terms of the formal logic, we have that, "for all i≠j in I, *almost surely* the event $Z_i \neq Z_j$ occurs", while "*almost surely it is false that* for all i≠j∈ I, the event $Z_i \neq Z_j$ occurs". (1.2.1)

LET p, q, r, s: i, j, I, Z

((#(p@q)<r)>%((s&p)@(s&q)))&~((#(p@q)<r)>%((s&p)@(s&q))) ;
FFFF FFFF FFFF FFFF (1.2.2)

It is natural to ask whether it is possible to revise the notion of *almost sure*ly so that its meaning remains unchanged for simple logical assertions such as $Z_i \neq Z_j$ but such that it commutes with quantification. ... Such a formalism would describe truth in a necessarily larger model of mathematics, one in which there are new outcomes to the random experiment which did not exist before the experiment was performed.

From a modern perspective, forcing provides a formalism for examining what occurs *almost surely* not only in probability spaces but also in a much more general setting than what is provided by our conventional notion of randomness. Forcing has proved extremely useful in developing and understanding of models of set theory and in determining what can and cannot be proved within the standard axiomatization of mathematics (which we will take to be ZFC). (1.3.1)

> **Remark 1.3.1:** The author assumes ZFC is an axiomatization of mathematics, which elsewhere we show otherwise.

In fact it is a heuristic of modern set theory that if a statement arises naturally in mathematics and is consistent, then its consistency can be established using forcing, possibly starting from a large cardinal hypothesis. (1.4.1)

> **Remark 1.4.1:** An heuristic is not a theorem but the hypothetical starting point of an hypothesis as evaluated by an iterative loop of trial-and-error.

The focus of this article ... is to demonstrate how the method of forcing can be used to p*rove theorem*s as opposed to *establish consistency result*s. Forcing itself concerns the study of adding *generic object*s to a model of set theory, resulting in a larger model of set theory. One of the key aspects of forcing is that it provides a formalism for studying what happens *almost surely* as the result of introducing a generic object. An analysis of this formalism sometimes leads to new results concerning the original model itself — results which are in fact independent of the model entirely. (1.5.1)

> **Remark 1.5.1:** The method of forcing injects itself onto a fiducial model as a larger abstraction which is then named differently as a generic model. However, the larger problem of this method is that forcing cannot be entirely separated from and fully independent of the original model as its basis.

Eqs. 1.1.2 and 1.2.2 as rendered are *no*t tautologous with the latter as a contradiction to mean it is not proved as a paradox. This means the two examples in the introduction to demonstrate the forcing method cannot be forced into proofs, hence refuting the method of forcing to produce larger truth values. What follows is that the forcing method is better suited for paraconsistent logics which we demonstrate elsewhere are *non* tautologous fragments of the universal logic VŁ4.