A theorem on Sum of Triple of Distinct Proper Fractions

Faisal Amin Yassein Abdelmohssin[1]

Abstract:

I claim that the sum of following distinct proper fractions $(\frac{1}{2},\frac{1}{3},\frac{1}{6})$ is the *only* triple of distinct proper fraction that sum to 1 (i.e. $\frac{1}{2}+\frac{1}{3}+\frac{1}{6}=1$).

MSC numbers:[2] 11Axx

Keywords: Elementary number theory

Introduction

I am motivated by my study of King James Version (KJV)[3] and the book of Sahih Bukhari: Volume 2, Book 21, Number 231[4], that the set of natural numbers $(1,2,3,6)$ has very interesting arithmetic properties, so does the sum of the fractions of the triple $(\frac{1}{2},\frac{1}{3},\frac{1}{6})$.

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[1] f.a.y.abdelmohssin@gmail.com

[2] https://cran.r-project.org/web/classifications/MSC.html
http://www.ams.org/msc/msc2010.html

[3] [62]Psalms 119:62 David said, "At *midnight* I will rise to give thanks unto thee....". As a prophet David knew the importance of *Midnight prayer*.

[4] The most beloved of actions:
Narrated Abdullah bin ‘Amr bin Al-‘As: “Allah’s Apostle told me, “The most beloved prayer to Allah is that of David and the most beloved fasts to Allah are those of David. He used to sleep for *half of the night* and then pray for *one third of the night* and again sleep for *its sixth part* and used to fast on alternate days.” – Sahih Bukhari: Volume 2, Book 21, Number 231.

Discussion:

Firstly, I show the arithmetic properties of the triple of the natural number $(1,2,3)$.

(i)Their sum:

$$1+2+3=6 \quad \textbf{(1)}$$

(ii)Their Multiplication:

$$1\times 2\times 3=6 \quad \textbf{(2)}$$

(iii) Factorial 3:

$$3!=3\times 2\times 1=6 \quad \textbf{(3)}$$

Theorem:

$(\frac{1}{2},\frac{1}{3},\frac{1}{6})$ is the *only* triple of distinct proper fraction that sum to 1

(i.e. $\frac{1}{2}+\frac{1}{3}+\frac{1}{6}=1$).

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