

Refutation of relevance logic R and models

© Copyright 2018 by Colin James III All rights reserved.

Abstract: We evaluate a definition and model formula for relevance logic R which are *not* tautologous. What follows is that logic R is a *non* tautologous fragment of the universal logic VL4.

We assume the method and apparatus of Meth8/VL4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET p, q, r: A, B, C;
 ~ Not, ¬; + Or, ∨, ∪; - Not Or; & And, ∧, ∩; \ Not And;
 > Imply, greater than, →, ↗, >, ⊃, ⊃, ⊃, ↘; < Not Imply, less than, ∈, <, ⊂, ⊄, ⊆, ⊇;
 = Equivalent, ≡, :=, ⇔, ↔, ≅ @ Not Equivalent, ≠;
 % possibility, for one or some, ∃, ∃, M; # necessity, for every or all, ∀, □, L;
 (z=z) T as tautology, T, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;
 (%z<#z) C as contingency, Δ, ordinal 1; (%z>#z) N as non-contingency, ∇, ordinal 2;
 ~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
 Note: For clarity we usually distribute quantifiers on each variable as designated.

From: en.wikipedia.org/wiki/Relevance_logic

System E in relevance logic adds this definition of

$$\Box A \text{ as } (A \rightarrow A) \rightarrow A. \tag{1.1.1}$$

$$\#p = ((p > p) > p); \quad \text{TNTN TNTN TNTN TNTN} \tag{1.1.2}$$

Remark 1.1. Eq. 1.1.2 is *not* tautologous. If Eq.1.1 is substituted back into the E axiom $\Box A \wedge \Box B \rightarrow \Box (A \wedge B)$, the result is a theorem but only by way of an injection of non-tautologous axiom definition.

"The conditional fragment of R is sound and complete with respect to the class of semilattice models. The logic with conjunction and disjunction is properly stronger than the conditional, conjunction, disjunction fragment of R. In particular, the formula

$$(A \rightarrow (B \vee C)) \wedge (B \rightarrow C) \rightarrow (A \rightarrow C) \tag{1.2.1}$$

is valid for the operational models but it is invalid in R."

$$(p > (q + r)) \& ((q > r) > (p > r)); \quad \text{TFTT TTTT TFTT TTTT} \tag{1.2.2}$$

Remark 1.2. Eq. 1.2 is *not* tautologous.

Eqs. 1.1.2 and 1.2.2 refute relevance logic.