Refutation of the unification type of simple symmetric modal logics

Abstract: We evaluate the definitions of two new modal connectives as box-plus and box-minus. Neither is tautologous and both are equivalent. This refutes the unification type of simple symmetric modal logics and implies it is a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪; - Not Or; & And, ∧, ∩; \ Not And;
\> Imply, greater than, →, ↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞;
= Equivalent, ≡, :=, ⟺, ↔, ≙; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;
(%z<#z) C as contingency, Δ, ordinal 1; (%z>#z) N as non-contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

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2 Syntax.
For all parameters p, we write "p^0" to mean "¬p" and we write "p^1" to mean "p".
Let ⊞and ⊟be the modal connectives defined as follows: (2.0)

Remark 2.0: We name the respective symbols ⊞ and ⊟as box-plus and box-minus.

$$⊞\phi ::= (p^0 \wedge q^0 \rightarrow \Box(p^1 \wedge q^0 \rightarrow \Box(p^0 \wedge q^1 \rightarrow \Box(p^0 \wedge q^0 \rightarrow \phi)))), \quad (2.1.1)$$

LET p, q, r: p^1=p, q^1=q, p^0=~p, q^0=~q, φ.

((~p&~q)>#((p&~q)>#((~p&q)>#((~p&~q)>t)))) = (p=p) ;
NTTT NTTT NTTT NTTT (2.1.2)

$$⊟\phi ::= (p^0 \wedge q^0 \rightarrow \Box(p^0 \wedge q^1 \rightarrow \Box(p^1 \wedge q^0 \rightarrow \Box(p^0 \wedge q^0 \rightarrow \phi)))). \quad (2.2.1)$$

((~p&~q)>#((~p&q)>#((p&~q)>#((~p&~q)>t)))) = (p=p) ;
NTTT NTTT NTTT NTTT (2.2.2)

Eqs. 2.1.2 and 2.2.2 are *not* tautologous and logically equivalent because the respective conjunctive clauses are identical. This refutes the unification type of simple symmetric modal logics.