**Refutation of prevarieties and quasivarieties of logic**



**Abstract:** We evaluate two papers by the same author group. For prevarieties, we test a theorem in M which is *not* tautologous. For quasivarieties, we test a definition and two theorems. The definition of a De Morgan monoid via an involution function is *not* tautologous. Theorems for the Dunn monoid and via Brouwerian (and Heyting) algebra are *not* tautologous. These results collectively refute prevarieties and quasivarieties in logic. What follows is that prevarieties and quasivarieties of logic are *non* tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪; - Not Or; & And, ∧, ∩, ·; \ Not And;
> Imply, greater than, →, ↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞;
= Equivalent, ≡, :=, ⟺, ↔, ≜, ≈; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;
(%z<#z) C as contingency, Δ, ordinal 1; (%z>#z) N as non-contingency, ∇, ordinal 2;
~( y < x) ( x ≤ y), ( x ⊆y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

From: Moraschini, T.; Raftery, J.G. (2019). On prevarieties of logic.
arxiv.org/pdf/1902.04160.pdf moraschini@cs.cas.cz, james.raftery@up.ac.za

Abstract: It is proved that every prevariety of algebras is categorically equivalent to a ‘prevariety of logic’, i.e., to the equivalent algebraic semantics of some sentential deductive system. This allows us to show that no nontrivial equation in the language ∧,∨,◦ holds in the congruence lattices of all members of every variety of logic, and that being a(pre)variety of logic is not a categorical property.

Let M be the matrix power K. ... [T]he following formula is valid in M:

$$(x\rightarrow y\approx\Box(x\rightarrow y)\&x\leftarrow y\approx\Box(x\leftarrow y))\Longleftrightarrow x\approx y \qquad (4.1)$$

LET p, q, r, s: *x*, *y*, *z*, *e*.

(((p>q)=#(p>q))&((p<q)=#(p<q)))=(p=q) ; NCCN NCCN NCCN NCCN (4.2)

**Remark 4.2:** Eq. 4.2 as rendered is *not* tautologous.
This refutes the prevariety of logic.

From: Moraschini, T.; Raftery, J.G.; Wannenburg, J.J. (2019).
Singly generated quasivarieties and residuated structures.
arxiv.org/pdf/1902.04159.pdf

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Definition 8.1.
A *De Morgan monoid* is an algebra $\boldsymbol{A} = \langle A;\cdot,\wedge,\vee,\neg, e\rangle$ comprising a distributive lattice $\langle A;\wedge,\vee\rangle$, a commutative monoid $\langle A;\cdot, e\rangle$ that is *square-increasing* (i.e., $\boldsymbol{A}$ satisfies $x\leq x^2 := x\cdot x$), and a function $\neg\colon A\to A$, called an *involution*, such that $\boldsymbol{A}$ satisfies $\neg\neg x \approx x$ and

$$x\cdot y\leq z \Longleftrightarrow x\cdot\neg z\leq\neg y \quad (8.1.1)$$

~(r<(p&q))=~(~q<(p&~r)) ; **F**TTT TT**F**T **F**TTT TT**F**T (8.1.2)

**Remark 8.1.2:** Eq. 8.1.2 as rendered is *not* tautologous. This refutes the definition of a De Morgan monoid via an involution function.

9. Dunn monoids and reflections
With respect to the derived operation $x\to y := \neg(x\cdot\neg y)$, every De Morgan monoid satisfies $\neg x\approx x\to f$ and

$$x\cdot y\leq z \Longleftrightarrow y\leq x\to z \text{ (the law of residuation).} \quad (9.6.1)$$

~(r<(p&q))=~((p>r)<q) ; **F**TTT TT**F**T **F**TTT TT**F**T (9.6.2)

**Remark 9.6.2**: Eq. 9.6.2 is *not* tautologous. We note that Eqs.8.1.2 and 9.6.2 produce the same truth table result.
This refutes Dunn monoids via the law of residuation.

10. Brouwerian algebras
Definition 10.1. A Dunn monoid is called a *Brouwerian algebra* if it satisfies $x\cdot y\approx x\wedge y$ (or equivalently, $x\leq e$), in which case it is identified with its $\to$, $\wedge$, $\vee$, $e$ reduct. A Heyting algebra is therefore just a Brouwerian algebra with a distinguished least element.

Mints [47] showed (in effect) that the variety BRA of *all* Brouwerian algebras is not SC, by proving that the following quasi-equation (not satisfied by BRA) is admissible in BRA:

$$x\to y\leq x\vee z \Rightarrow ((x\to y)\to x)\vee((x\to y)\to z)\approx e. \quad (10.1)$$

**Remark 10.1:** For our purpose in testing, we ignore the trailing equivalent.

~((p+r)<(p>q))>(((p>q)>p)+((p>q)>r)) ; **F**T**F**T TTTT **F**T**F**T TTTT (10.2)

**Remark 10.2**: Eq. 10.2 is *not* tautologous.
We note that Eqs. 9.6.2 and 10.2 produce the same truth table results.
This refutes Brouwerian *and* Heyting algebra.

Eqs. 4.2, 8.1.2, 9.6.2, and 10.2 collectively refute prevarieties and quasivarieties in logic, and in the process refute the De Morgan and Dunn monoids, and Brouwerian and Heyting algebras.