Refutation of Peirce's abduction and induction, and confirmation of deduction

Abstract: We evaluate definitions of C.S. Peirce for abduction, induction and deduction: all are inversions of the same sentence. However, when the connectives are changed to implication, abduction and induction are not tautologous, leaving deduction as the only form of tautologous inference in logic.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞;
= Equivalent, ≡, :=, ⟺, ↔, ≜, ≈; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;
(%z<#z) C as contingency, Δ, ordinal 1; (%z>#z) N as non-contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

From: iep.utm.edu/peir-log/

C.S. Peirce originally defined the three forms of inference in logic as:

Abduction: (Q is S) and (Q is P) imply (S is P) (1.1.1)

LET p, q, s: P, Q, S.

((q=s)&(q=p))>(s=p) ; TTTT TTTT TTTT TTTT (1.1.2)

Induction: (S is Q) and (P is Q) imply (S is P) (2.1.1)

((s=q)&(p=q))>(s=p) ; TTTT TTTT TTTT TTTT (2.1.2)

Deduction: (S is Q) and (Q is P) imply (S is P) (3.1.1)

((s=q)&(q=p))>(s=p) ; TTTT TTTT TTTT TTTT (3.1.2)

Peirce described Eqs. 1-3 as inversions of the same.

Remark: If the word "is" is taken to mean the word "implies" then the connective = is replaced with the connective > below.

Abduction: (Q implies S) and (Q implies P) imply (S implies P) (1.2.1)

((q>s)&(q>p))>(s>p) ; TTTT TTTT **F**TTT **F**TTT (1.2.2)

Induction: (S implies Q) and (P implies Q) imply (S implies P) (2.2.1)

((s>q)&(p>q))>(s>p) ; TTTT TTTT TT**F**T TT**F**T (2.2.2)

Deduction: (S implies Q) and (Q implies P) imply (S implies P) (3.2.1)

((s>q)&(q>p))>(s>p) ; TTTT TTTT TTTT TTTT (3.2.2)

Eqs. 1.2.2-2.2.2 as rendered for abduction and induction are *not* tautologous, but Eq. 3.2.2 is tautologous. This means that abduction and induction are not inversions of deduction, leaving deduction as the only form of tautologous inference in logic.