Refutation of provability of consistency with Rosser's theorem

Abstract: We evaluate provability of consistency with Rosser's theorem. A trivial theorem is found in the abstract, but a rule of necessitation and Rosser's theorem are *not* tautologous. While consistency is likely provable, the instant approach is refuted and thus not vindicating Hilbert.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞;
= Equivalent, ≡, :=, ⟺, ↔, ≜, ≈; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, ∅, Null, ⊥, zero;
(%z<#z) `C` as contingency, Δ, ordinal 1; (%z>#z) `N` as non-contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

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F is constructively false iff PA proves 'for each x, there is a proof that x is not a proof of F.' (0.1.1)

LET q, r: F, x

~(#r>(q=(q=q)))>(q=(q@q)) ; `TTTT TTTT TTTT TTTT` (0.1.2)

[F]or any PA-derivation S we find a finitary proof that S does not contain 0=1 (0.2.1)

LET s: S

~(#(s=s)>((s=s)=(%s>#s))) = (s=s) ; `CCCC CCCC CCCC CCCC` (0.2.2)

Remark 2.1: If by "0=1" the intention is **`F`**=`T`, then Eq. 0.2.1 is rendered as:
~(#(s=s)>((s=s)=(s=s))) = (s=s) ; **`FFFF FFFF FFFF FFFF`** (0.2.3)

[A]ny finite sequence S of formulas is not a derivation of a contradiction. [Claim 1] (1.1.1)

~(%(s@s)>#s) = (s=s) ; `CCCC CCCC CCCC CCCC` (1.1.2)

Remark 1.1: We map "a contradiction" to mean at least one contradiction.
That strengthens Eq. 1.1.1 from a contradiction to a falsity.

[T]here is a finitary proof p(S) that S is not a derivation of a contradiction. [Claim 2] (1.2.1)

LET p: p.

%(p&s)>~(%(s@s)>s) ; `TTTT TTTT NFNF NFNF` (1.2.2)

Rule of Necessitation: $\frac{\vdash F}{\vdash \Box F}$ (2.1)

LET p: F.

q>#q ; `TTNN TTNN TTNN TTNN` (2.2)

Remark 2.1: This rule potentially taints the remaining assertions.

Rosser sentence R and its negation ¬R are both constructively false.
The proof of Rosser's Theorem is syntactic and can be formalized in PA

$$PA \vdash \neg\Box\bot\rightarrow(\neg\Box R \wedge \neg\Box\neg R). \quad (4.1)$$

LET r: R.
~(#(r@r)=(r=r))>(~#r&~#~r) ; `CCCC CCCC CCCC CCCC` (4.2)

Remark 4.2: Rosser's theorem as rendered in Eq. 4.2 is *not* tautologous, *not* contradictory per se, but is a falsity.