Quantum States and Energy Levels in Hydrogen Atom

Daniele Sasso

Abstract

In this paper we will calculate energy levels of electron relative to hydrogen atom making use of the four quantum numbers of the deterministic quantum model. Calculations prove in a few situations quantum states with different quantum numbers can have the same value of total energy and a few sub-levels of a more external level can precede sub-levels of more internal levels. Calculations prove also the electron in the fundamental state 1s1, in order to jump to the state 5q10 needs a photon with energy E=13.08 eV= $20.93 \cdot 10^{-19}$ J, that is in the ultraviolet band.

Introduction

Energy values of electron inside atom are given by the deterministic quantum model[1] represented in Deterministic Quantum Physics[2] by the following relation

$$E_{nkjs} = -\frac{2Z^2Rhc}{n^2}\left(1 - \frac{k^2}{2n^2}\right)\left(1 - \frac{1}{2}\frac{\alpha^2Z^2(j-s)^2}{n^4}\right) \qquad (1)$$

in which

n = 1, 2 , ...	quantum number of level
k = 1, 2 ,....... n	quantum number of sub-level
j = ±1, ±2, ...±k	quantum number of orbital momentum
$s = \frac{\lvert j \rvert}{2}$	quantum number of spin

The quantity Z in (1) represents the atomic number and hence for any Z the (1) represents the atomic model for any chemical element. The (1) gives levels of total energy and for Z=1 it gives levels of total energy of hydrogen atom. Sub-levels of any level are normally indicated with conventional letters that are used also in this paper:

- the first sub-level is identified by the letter “ s ” that represents k=1
- the second sub-level is identified by the letter “ p ” thar represents k=2
- the third sub-level is identified by the letter “ d ” that represents k=3
- the fourth sub-level is identified by the letter “ f ” that represents k=4
- the fifth sub-level is identified by the letter “ q “ that represents k=5.

1. **Calculation of energy levels in hydrogen atom**

Assuming Z=1, from the (1) we are able to calculate energy levels of electron inside hydrogen atom for different values of quantum numbers.
Firstly we observe constants in (1) have the following values in the International System (IS) of measurement:

a. Rydberg's constant $R = \frac{e^4 m_o}{8\varepsilon_o^2 c h^3} = 10973731 \; m^{-1}$ (2)

b. constant of fine structure (or Lamb's constant) $\alpha = \frac{e^2}{2\varepsilon_o c h} = \frac{1}{137.036}$ (3)

In (1) considering Z=1, the constant quantity D=2Rhc has the following value

$$D = 2Rhc = 436233021.343 \cdot 10^{-26} \; J = 27.2645638339 \; eV \qquad (4)$$

in which

$$1 \; eV = 1.6 \cdot 10^{-19} \; J \qquad (5)$$

Hydrogen atom has Z=1 and hence it has only one orbital electron that occupies normally the fundamental level characterized by the following quantum numbers: n=1, k=1, j=1, s=1/2.

1.1 Quantum level n=1

The level n=1 represents the fundamental level of atom and it has always only two quantum states characterized by quantum numbers n=1, k=1, j=±1, s=1/2.
The two states are indicated with 1s1 and 1s2, where the first number indicates the level, the letter s indicates the conventional name of the first sub-level and the second number indicates the number of electron in the sub-level.

1.1.1 k=1 , sub-level 1s

Energy level 1s1 for hydrogen atom (Z=1), that is also the effective level of atom occupied by the unique electron of hydrogen, for j=1 and s=1/2, is given by

$$E_{1s1} = - D \frac{1}{2}\left(1 - \frac{\alpha^2}{8}\right) = - 13.63219 \; eV \qquad (6)$$

Energy given by (6) represents energy of the unique electron of hydrogen atom. It constitutes also ionization energy of hydrogen atom that is the necessary energy in order

to extract electron from atom and to ionize it. The outcome of calculation is in good concordance with experimental results.
Energy of level 1s2, defined by quantum numbers n=1, k=1, j=-1, s=1/2 is given by

$$E_{1s2} = - D \frac{1}{2} \left(1 - \frac{9\alpha^2}{8} \right) = - 13.63147 \text{ eV} \qquad (7)$$

1.2 Quantum level n=2

The level n=2 of energy is constituted by two sub-levels s and p, everyone containing 4 quantum states for an amount of 8 quantum states in the second level.
In the second level quantum numbers assume the following values: n=2, k=1,2 j=±1,±2, s=1/2,1

1.2.1 k=1 , sub-level 2s

Energy of the state that has quantum numbers n=2, k=1, j=1, s=1/2 is

$$E_{2s1} = - \frac{D}{4} \frac{7}{8} \left(1 - \frac{\alpha^2}{128} \right) = - 5.96412 \text{ eV} \qquad (8)$$

Energy of the state with quantum numbers n=2, k=1, j=2, s=1 is

$$E_{2s2} = - \frac{D}{4} \frac{7}{8} \left(1 - \frac{\alpha^2}{32} \right) = - 5.96411 \text{ eV} \qquad (9)$$

Energy of the state with quantum numbers n=2, k=1, j=-1, s=1/2 is

$$E_{2s3} = - \frac{D}{4} \frac{7}{8} \left(1 - \frac{9\alpha^2}{128} \right) = - 5.96410 \text{ eV} \qquad (10)$$

The state with quantum numbers n=2, k=1, j=-2, s=1 has an energy

$$E_{2s4} = - \frac{D}{4} \frac{7}{8} \left(1 - \frac{9\alpha^2}{32} \right) = - 5.96403 \text{ eV} \qquad (11)$$

1.2.2 k=2 sub-level 2p

Energy of the state that has quantum numbers n=2, k=2, j=1, s=1/2 is given by

$$E_{2p1} = - \frac{D}{4} \frac{1}{2} \left(1 - \frac{\alpha^2}{128} \right) = - 3.408069 \text{ eV} \qquad (12)$$

The state characterized by quantum numbers n=2, k=2, j=2, s=1 is

$$E_{2p2} = -\frac{D}{4}\,\frac{1}{2}\left(1 - \frac{\alpha^2}{32}\right) = -3.408065 \text{ eV} \qquad (13)$$

Energy of the state with quantum numbers n=2, k=2, j=-1, s=1/2 is

$$E_{2p3} = -\frac{D}{4}\,\frac{1}{2}\left(1 - \frac{9\alpha^2}{128}\right) = -3.408058 \text{ eV} \qquad (14)$$

Energy of the state that has quantum numbers n=2, k=2, j=-2, s=1 is

$$E_{2p4} = -\frac{D}{4}\,\frac{1}{2}\left(1 - \frac{9\alpha^2}{32}\right) = -3.408019 \text{ eV} \qquad (15)$$

1.3 Quantum level n=3

The level n=3 is composed of 3 sub-levels s, p and d, everyone containing 6 quantum states for an amount of 18 quantum states.
In the third level quantum numbers assume the following values n=3, k=1,2,3 j=±1,±2,±3 s=1/2,1,3/2

1.3.1 k=1 , sub-level 3s

Energy of the quantum state with quantum numbers n=3, k=1, j=1, s=1/2 is

$$E_{3s1} = -\frac{D}{9}\,\frac{17}{18}\left(1 - \frac{\alpha^2}{648}\right) = -2.8610952978 \text{ eV} \qquad (16)$$

The quantum state with quantum numbers n=3, k=1, j=2, s=1 has the energy

$$E_{3s2} = -\frac{D}{9}\,\frac{17}{18}\left(1 - \frac{\alpha^2}{162}\right) = -2.8610952643 \text{ eV} \qquad (17)$$

Energy of the quantum state that has quantum numbers n=3, k=1, j=3, s=3/2 is

$$E_{3s3} = -\frac{D}{9}\,\frac{17}{18}\left(1 - \frac{9\,\alpha^2}{648}\right) = -2.86109408865 \text{ eV} \qquad (18)$$

Energy of the quantum state n=3, k=1, j=-1, s=1/2 is given by

$$E_{3s4} = -\frac{D}{9}\,\frac{17}{18}\left(1 - \frac{9\,\alpha^2}{648}\right) = -2.86109408865 \text{ eV} \qquad (19)$$

Quantum state n=3, k=1, j=-2, s=1 , energy is

$$E_{3s5} = -\frac{D}{9}\frac{17}{18}\left(1-\frac{9\,\alpha^2}{162}\right) = -\,2.86109408865\text{ eV} \qquad (20)$$

Quantum state n=3, k=1, j=-3, s=3/2 , energy is

$$E_{3s6} = -\frac{D}{9}\frac{17}{18}\left(1-\frac{81\alpha^2}{648}\right) = -\,2.86107716014\text{ eV} \qquad (21)$$

1.3.2 k=2 , sub-level 3p

Quantum state n=3, k=2, j=1, s=1/2 , energy is given by

$$E_{3p1} = -\frac{D}{9}\frac{14}{18}\left(1-\frac{\alpha^2}{648}\right) = -\,2.3561966809\text{ eV} \qquad (22)$$

Quantum state n=3, k=2, j=2, s=1 , energy is

$$E_{3p2} = -\frac{D}{9}\frac{14}{18}\left(1-\frac{\alpha^2}{162}\right) = -\,2.35619610001\text{ eV} \qquad (23)$$

Quantum state n=3, k=2, j=3, s=3/2 , energy is

$$E_{3p3} = -\frac{D}{9}\frac{14}{18}\left(1-\frac{9\,\alpha^2}{648}\right) = -\,2.35619513187\text{ eV} \qquad (24)$$

Quantum state n=3, k=2, j=-1, s=1/2 , energy is

$$E_{3p4} = -\frac{D}{9}\frac{14}{18}\left(1-\frac{9\,\alpha^2}{648}\right) = -\,2.35619513187\text{ eV} \qquad (25)$$

Quantum state n=3, k=2, j=-2, s=1, energy is given by

$$E_{3p5} = -\frac{D}{9}\frac{14}{18}\left(1-\frac{9\,\alpha^2}{162}\right) = -\,2.35618990395\text{ eV} \qquad (26)$$

Quantum state n=3, k=2, j=-3, s=3/2 , energy is

$$E_{3p6} = -\frac{D}{9}\frac{14}{18}\left(1-\frac{81\,\alpha^2}{648}\right) = -\,2.35618119068\text{ eV} \qquad (27)$$

1.3.3 k=3 , sub-level 3d

Quantum state n=3, k=3, j=1, s=1/2 , energy is given by

$$E_{3d1} = -\frac{D}{9}\frac{1}{2}\left(1 - \frac{\alpha^2}{648}\right) = -1.5146978663 \text{ eV} \qquad (28)$$

Quantum state n=3, k=3, j=2, s=1 , energy is

$$E_{3d2} = -\frac{D}{9}\frac{1}{2}\left(1 - \frac{\alpha^2}{162}\right) = -1.51469749287 \text{ eV} \qquad (29)$$

Quantum state n=3, k=3, j=3, s=3/2 , energy is

$$E_{3d3} = -\frac{D}{9}\frac{1}{2}\left(1 - \frac{9\alpha^2}{648}\right) = -1.5146968705 \text{ eV} \qquad (30)$$

Quantum state n=3, k=3, j=-1, s=1/2 , energy is given by

$$E_{3d4} = -\frac{D}{9}\frac{1}{2}\left(1 - \frac{9\alpha^2}{648}\right) = -1.5146968705 \text{ eV} \qquad (31)$$

Quantum state n=3, k=3, j=-2, s=1 , energy is

$$E_{3d5} = -\frac{D}{9}\frac{1}{2}\left(1 - \frac{9\alpha^2}{162}\right) = -1.51469350968 \text{ eV} \qquad (32)$$

Quantum state n=3, k=3, j=-3, s=3/2 , energy is

$$E_{3d6} = -\frac{D}{9}\frac{1}{2}\left(1 - \frac{81\alpha^2}{648}\right) = -1.51468790831 \text{ eV} \qquad (33)$$

1.4 Quantum level n=4

The level n=4 is composed of 4 sub-levels s, p, d and f, everyone is able to contain at most 8 quantum states for a total of 32 quantum states.
In the level 4 quantum numbers assume the following values n=4, k=1,2,3,4 j=±1,±2,±3,±4 s=1/2,1,3/2,2

1.4.1 k=1 , sub-level 4s

The state with quantum numbers n=4, k=1, j=1, s=1/2 has energy

$$E_{4s1} = -\frac{D}{16}\,\frac{31}{32}\left(1 - \frac{\alpha^2}{2048}\right) = -\,1.65078409545 \text{ eV} \qquad (34)$$

Energy of the quantum state n=4, k=1, j=2, s=1 is

$$E_{4s2} = -\frac{D}{16}\,\frac{31}{32}\left(1 - \frac{\alpha^2}{512}\right) = -\,1.65078396669 \text{ eV} \qquad (35)$$

Quantum state n=4, k=1, j=3, s=3/2 , energy is

$$E_{4s3} = -\frac{D}{16}\,\frac{31}{32}\left(1 - \frac{9\,\alpha^2}{2048}\right) = -\,1.65078375207 \text{ eV} \qquad (36)$$

Quantum state n=4, k=1, j=4, s=2 , energy is given by

$$E_{4s4} = -\frac{D}{16}\,\frac{31}{32}\left(1 - \frac{4\,\alpha^2}{512}\right) = -\,1.6507834516 \text{ eV} \qquad (37)$$

Energy of the state n=4, k=1, j=-1, s=1/2 is

$$E_{4s5} = -\frac{D}{16}\,\frac{31}{32}\left(1 - \frac{9\,\alpha^2}{2048}\right) = -\,1.65078375207 \text{ eV} \qquad (38)$$

Quantum state n=4, k=1, j=-2, s=1 , energy is

$$E_{4s6} = -\frac{D}{16}\,\frac{31}{32}\left(1 - \frac{9\,\alpha^2}{512}\right) = -\,1.65078259315 \text{ eV} \qquad (39)$$

Quantum state n=4, k=1, j=-3, s=3/2 , energy is

$$E_{4s7} = -\frac{D}{16}\,\frac{31}{32}\left(1 - \frac{81\,\alpha^2}{2048}\right) = -\,1.6507806616 \text{ eV} \qquad (40)$$

Quantum state n=4, k=1, j=-4, s=2 , energy is given by

$$E_{4s8} = -\frac{D}{16}\,\frac{31}{32}\left(1 - \frac{36\,\alpha^2}{512}\right) = -\,1.65077795745 \text{ eV} \qquad (41)$$

1.4.2 k=2 , sub-level 4p

The state with quantum numbers n=4, k=2, j=1, s=1/2 has energy

$$E_{4p1} = -\frac{D}{16}\,\frac{7}{8}\left(1 - \frac{\alpha^2}{2048}\right) = -\,1.49103079589 \text{ eV} \qquad (42)$$

Quantum state n=4, k=2, j=2, s=1 , energy is

$$E_{4p2} = - \frac{D}{16}\,\frac{7}{8}\left(1 - \frac{\alpha^2}{512}\right) = - 1.49103067959 \text{ eV} \qquad (43)$$

Energy of the state n=4, k=2, j=3, s=3/2 is given by

$$E_{4p3} = - \frac{D}{16}\,\frac{7}{8}\left(1 - \frac{9\,\alpha^2}{2048}\right) = - 1.49103048574 \text{ eV} \qquad (44)$$

Quantum state n=4, k=2, j=4, s=2 , energy is

$$E_{4p4} = - \frac{D}{16}\,\frac{7}{8}\left(1 - \frac{4\,\alpha^2}{512}\right) = - 1.49103021436 \text{ eV} \qquad (45)$$

Quantum state n=4, k=2, j=-1, s=1/2 , energy is

$$E_{4p5} = - \frac{D}{16}\,\frac{7}{8}\left(1 - \frac{9\,\alpha^2}{2048}\right) = - 1.49103048574 \text{ eV} \qquad (46)$$

Quantum state n=4, k=2, j=-2, s=1 , energy is

$$E_{4p6} = - \frac{D}{16}\,\frac{7}{8}\left(1 - \frac{9\,\alpha^2}{512}\right) = - 1.49102943897 \text{ eV} \qquad (47)$$

Energy of the quantum state n=4, k=2, j=-3, s=3/2 is

$$E_{4p7} = - \frac{D}{16}\,\frac{7}{8}\left(1 - \frac{81\alpha^2}{2048}\right) = - 1.49102769435 \text{ eV} \qquad (48)$$

Quantum state n=4, k=2, j=-4, s=2 , energy is given by

$$E_{4p8} = - \frac{D}{16}\,\frac{7}{8}\left(1 - \frac{36\,\alpha^2}{512}\right) = - 1.49102525188 \text{ eV} \qquad (49)$$

1.4.3 k=3 , sub-level 4d

Quantum state n=4, k=3, j=1, s=1/2 , energy is

$$E_{4d1} = - \frac{D}{16}\,\frac{23}{32}\left(1 - \frac{\alpha^2}{2048}\right) = - 1.22477529662 \text{ eV} \qquad (50)$$

Quantum state n=4, k=3, j=2, s=1 , energy is

$$E_{4d2} = -\frac{D}{16}\frac{23}{32}\left(1 - \frac{\alpha^2}{512}\right) = -1.22477520109 \text{ eV} \qquad (51)$$

Quantum state n=4, k=3, j=3, s=3/2 , energy is

$$E_{4d3} = -\frac{D}{16}\frac{23}{32}\left(1 - \frac{9\alpha^2}{2048}\right) = -1.22477504185 \text{ eV} \qquad (52)$$

Energy of the quantum state n=4, k=3, j=4, s=2 is given by

$$E_{4d4} = -\frac{D}{16}\frac{23}{32}\left(1 - \frac{4\alpha^2}{512}\right) = -1.22477520109 \text{ eV} \qquad (53)$$

Energy of the quantum state n=4, k=3, j=-1, s=1/2 is

$$E_{4d5} = -\frac{D}{16}\frac{23}{32}\left(1 - \frac{9\alpha^2}{2048}\right) = -1.22477504185 \text{ eV} \qquad (54)$$

Energy of the quantum state n=4, k=3, j=-2, s=1 is

$$E_{4d6} = -\frac{D}{16}\frac{23}{32}\left(1 - \frac{9\alpha^2}{512}\right) = -1.22477418201 \text{ eV} \qquad (55)$$

Quantum state n=4, k=3, j=-3, s=3/2 has energy

$$E_{4d7} = -\frac{D}{16}\frac{23}{32}\left(1 - \frac{81\alpha^2}{2048}\right) = -1.22477274891 \text{ eV} \qquad (56)$$

Quantum state n=4, k=3, j=-4, s=2 has energy

$$E_{4d8} = -\frac{D}{16}\frac{23}{32}\left(1 - \frac{36\alpha^2}{512}\right) = -1.22477074262 \text{ eV} \qquad (57)$$

1.4.4 k=4 , sub-level 4f

Energy of the quantum state n=4, k=4, j=1, s=1/2 is

$$E_{4f1} = -\frac{D}{16}\frac{1}{2}\left(1 - \frac{\alpha^2}{2048}\right) = -0.85201759765 \text{ eV} \qquad (58)$$

Energy of the quantum state n=4, k=4, j=2, s=1 is

$$E_{4f2} = -\frac{D}{16}\frac{1}{2}\left(1 - \frac{\alpha^2}{512}\right) = -0.85201753119 \text{ eV} \qquad (59)$$

Quantum state n=4, k=4, j=3, s=3/2 has energy

$$E_{4f3} = -\frac{D}{16}\frac{1}{2}\left(1 - \frac{9\alpha^2}{2048}\right) = -0.85201742042 \text{ eV} \qquad (60)$$

Quantum state n=4, k=4, j=4, s=2 has energy

$$E_{4f4} = -\frac{D}{16}\frac{1}{2}\left(1 - \frac{4\,\alpha^2}{512}\right) = -0.85201726535 \text{ eV} \qquad (61)$$

Quantum state n=4, k=4, j=-1, s=1/2 has energy

$$E_{4f5} = -\frac{D}{16}\frac{1}{2}\left(1 - \frac{9\,\alpha^2}{2048}\right) = -0.85201742042 \text{ eV} \qquad (62)$$

Energy of the quantum state n=4, k=4, j=-2, s=1 is

$$E_{4f6} = -\frac{D}{16}\frac{1}{2}\left(1 - \frac{9\,\alpha^2}{512}\right) = -0.85201682227 \text{ eV} \qquad (63)$$

Energy of the quantum state n=4, k=4, j=-3, s=3/2 is given by

$$E_{4f7} = -\frac{D}{16}\frac{1}{2}\left(1 - \frac{81\alpha^2}{2048}\right) = -0.85201582534 \text{ eV} \qquad (64)$$

Energy of the quantum state n=4, k=4, j=-4, s=2 is given by

$$E_{4f8} = -\frac{D}{16}\frac{1}{2}\left(1 - \frac{36\alpha^2}{512}\right) = -0.85201442965 \text{ eV} \qquad (65)$$

1.5 Quantum level n=5

The level n=5 is composed of 5 sub-levels s, p, d, f and q, everyone with 10 quantum states for a total of 50 quantum states.
In the level 5 quantum numbers assume the following values: n=5, k=1,2,3,4,5 j=±1,±2,±3,±4,±5 s=1/2,1,3/2,2,5/2

1.5.1 k=1 , sub-level 5s

Energy of the quantum state n=5, k=1, j=1, s=1/2 is given by

$$E_{5s1} = -\frac{D}{25}\frac{49}{50}\left(1 - \frac{\alpha^2}{5000}\right) = -1.0687708909 \text{ eV} \qquad (66)$$

Energy of the quantum state n=5, k=1, j=2, s=1 is given by

$$E_{5s2} = -\frac{D}{25}\frac{49}{50}\left(1-\frac{\alpha^2}{1250}\right) = -1.06877085675 \text{ eV} \qquad (67)$$

Energy of the state with quantum numbers n=5, k=1, j=3, s=3/2 is

$$E_{5s3} = -\frac{D}{25}\frac{49}{50}\left(1-\frac{9\alpha^2}{5000}\right) = -1.06877079983 \text{ eV} \qquad (68)$$

Quantum state n=5, k=1, j=4, s=2 has energy

$$E_{5s4} = -\frac{D}{25}\frac{49}{50}\left(1-\frac{4\alpha^2}{1250}\right) = -1.06877072016 \text{ eV} \qquad (69)$$

Quantum state n=5, k=1, j=5, s=5/2 has energy

$$E_{5s5} = -\frac{D}{25}\frac{49}{50}\left(1-\frac{25\alpha^2}{5000}\right) = -1.06877061772 \text{ eV} \qquad (70)$$

Quantum state n=5, k=1, j=-1, s=1/2 has energy

$$E_{5s6} = -\frac{D}{25}\frac{49}{50}\left(1-\frac{9\alpha^2}{5000}\right) = -1.06877079983 \text{ eV} \qquad (71)$$

Quantum state n=5, k=1, j=-2, s=1 has energy

$$E_{5s7} = -\frac{D}{25}\frac{49}{50}\left(1-\frac{\alpha^2}{1250}\right) = -1.06877085675 \text{ eV} \qquad (72)$$

Quantum state n=5, k=1, j=-3, s=3/2 has energy

$$E_{5s8} = -\frac{D}{25}\frac{49}{50}\left(1-\frac{81\alpha^2}{5000}\right) = -1.06876998028 \text{ eV} \qquad (73)$$

Energy of the quantum state n=5, k=1, j=-4, s=2 is given by

$$E_{5s9} = -\frac{D}{25}\frac{49}{50}\left(1-\frac{36\alpha^2}{1250}\right) = -1.06876926318 \text{ eV} \qquad (74)$$

Energy of the quantum state n=5, k=1, j=-5, s=5/2 is

$$E_{5s10} = -\frac{D}{25}\frac{49}{50}\left(1-\frac{225\alpha^2}{5000}\right) = -1.06876834117 \text{ eV} \qquad (75)$$

1.5.2 k=2 , sub-level 5p

Energy of the quantum state n=5, k=2, j=1, s=1/2 is

$$E_{5p1} = -\frac{D}{25}\frac{46}{50}\left(1 - \frac{\alpha^2}{5000}\right) = -1.00333593839 \text{ eV} \tag{76}$$

Energy of the quantum state n=5, k=2, j=2, s=1 is

$$E_{5p2} = -\frac{D}{25}\frac{46}{50}\left(1 - \frac{\alpha^2}{1250}\right) = -1.00333590628 \text{ eV} \tag{77}$$

Energy of the quantum state n=5, k=2, j=3, s=3/2 is

$$E_{5p3} = -\frac{D}{25}\frac{46}{50}\left(1 - \frac{9\alpha^2}{5000}\right) = -1.00333585284 \text{ eV} \tag{78}$$

Quantum state n=5, k=2, j=4, s=2 has energy

$$E_{5p4} = -\frac{D}{25}\frac{46}{50}\left(1 - \frac{2\alpha^2}{625}\right) = -1.00333577786 \text{ eV} \tag{79}$$

Quantum state n=5, k=2, j=5, s=5/2 has energy

$$E_{5p5} = -\frac{D}{25}\frac{46}{50}\left(1 - \frac{25\alpha^2}{5000}\right) = -1.00333568189 \text{ eV} \tag{80}$$

Quantum state n=5, k=2, j=-1, s=1/2 has energy

$$E_{5p6} = -\frac{D}{25}\frac{46}{50}\left(1 - \frac{9\alpha^2}{5000}\right) = -1.00333585284 \text{ eV} \tag{81}$$

The state with quantum numbers n=5, k=2, j=-2, s=1 has energy

$$E_{5p7} = -\frac{D}{25}\frac{46}{50}\left(1 - \frac{9\alpha^2}{1250}\right) = -1.00333556438 \text{ eV} \tag{82}$$

The state with quantum numbers n=5, k=2, j=-3, s=3/2 has energy

$$E_{5p8} = -\frac{D}{25}\frac{46}{50}\left(1 - \frac{81\alpha^2}{5000}\right) = -1.00333508343 \text{ eV} \tag{83}$$

The state with quantum numbers n=5, k=2, j=-4, s=2 has energy

$$E_{5p9} = -\frac{D}{25}\frac{46}{50}\left(1 - \frac{36\,\alpha^2}{1250}\right) = -1.00333441027 \text{ eV} \qquad (84)$$

Quantum state n=5, k=2, j=-5, s=5/2 has energy

$$E_{5p10} = -\frac{D}{25}\frac{46}{50}\left(1 - \frac{225\,\alpha^2}{5000}\right) = -1.00333354462 \text{ eV} \qquad (85)$$

1.5.3 k=3 , sub-level 5d

Energy of the quantum state n=5, k=3, j=1, s=1/2 is given by

$$E_{5d1} = -\frac{D}{25}\frac{41}{50}\left(1 - \frac{\alpha^2}{5000}\right) = -0.89427745545\text{eV} \qquad (86)$$

Energy of the quantum state n=5, k=3, j=2, s=1 is given by

$$E_{5d2} = -\frac{D}{25}\frac{41}{50}\left(1 - \frac{\alpha^2}{1250}\right) = -0.89427765558 \text{ eV} \qquad (87)$$

Energy of the quantum state n=5, k=3, j=3, s=3/2 is

$$E_{5d3} = -\frac{D}{25}\frac{41}{50}\left(1 - \frac{9\,\alpha^2}{5000}\right) = -0.89427760786 \text{ eV} \qquad (88)$$

The quantum state n=5, k=3, j=4, s=2 has energy

$$E_{5d4} = -\frac{D}{25}\frac{41}{50}\left(1 - \frac{2\,\alpha^2}{625}\right) = -0.89427754134 \text{ eV} \qquad (89)$$

The quantum state n=5, k=3, j=5, s=5/2 has energy

$$E_{5d5} = -\frac{D}{25}\frac{41}{50}\left(1 - \frac{25\,\alpha^2}{5000}\right) = -0.89427745545 \text{ eV} \qquad (90)$$

The quantum state n=5, k=3, j=-1, s=1/2 has energy

$$E_{5d6} = -\frac{D}{25}\frac{41}{50}\left(1 - \frac{9\,\alpha^2}{5000}\right) = -0.89427760786 \text{ eV} \qquad (91)$$

Quantum state n=5, k=3, j=-2, s=1 has energy

$$E_{5d7} = -\frac{D}{25}\frac{41}{50}\left(1 - \frac{9\,\alpha^2}{1250}\right) = -0.89427735076 \text{ eV} \qquad (92)$$

The state with quantum numbers n=5, k=3, j=-3, s=3/2 has energy

$$E_{5d8} = -\frac{D}{25}\frac{41}{50}\left(1 - \frac{81\,\alpha^2}{5000}\right) = -\,0.89427692216 \text{ eV} \qquad (93)$$

Quantum state n=5, k=3, j=-4, s=2 has energy

$$E_{5d9} = -\frac{D}{25}\frac{41}{50}\left(1 - \frac{36\,\alpha^2}{1250}\right) = -\,0.89427632207 \text{ eV} \qquad (94)$$

Quantum state n=5, k=3, j=-5, s=5/2 has energy

$$E_{5d10} = -\frac{D}{25}\frac{41}{50}\left(1 - \frac{225\,\alpha^2}{5000}\right) = -\,0.89427555075 \text{ eV} \qquad (95)$$

1.5.4 k=4 , sub-level 5f

The quantum state n=5, k=4, j=1, s=1/2 has energy

$$E_{5f1} = -\frac{D}{25}\frac{34}{50}\left(1 - \frac{\alpha^2}{5000}\right) = -\,0.74159612837 \text{ eV} \qquad (96)$$

The quantum state n=5, k=4, j=2, s=1 has energy

$$E_{5f2} = -\frac{D}{25}\frac{34}{50}\left(1 - \frac{\alpha^2}{1250}\right) = -\,0.74159610465 \text{ eV} \qquad (97)$$

Energy of the quantum state n=5, k=4, j=3, s=3/2 is given by

$$E_{5f3} = -\frac{D}{25}\frac{34}{50}\left(1 - \frac{9\,\alpha^2}{5000}\right) = -\,0.74159606512 \text{ eV} \qquad (98)$$

Energy of the quantum state n=5, k=4, j=4, s=2 is given by

$$E_{5f4} = -\frac{D}{25}\frac{34}{50}\left(1 - \frac{2\,\alpha^2}{625}\right) = -\,0.74159600977 \text{ eV} \qquad (99)$$

Energy of the quantum state n=5, k=4, j=5, s=5/2 is given by

$$E_{5f5} = -\frac{D}{25}\frac{34}{50}\left(1 - \frac{25\,\alpha^2}{5000}\right) = -\,0.74159593861 \text{ eV} \qquad (100)$$

Energy of the quantum state n=5, k=4, j=-1, s=1/2 is

$$E_{5f6} = -\frac{D}{25}\frac{34}{50}\left(1 - \frac{9\,\alpha^2}{5000}\right) = -\,0.74159606512 \text{ eV} \qquad (101)$$

Energy of the quantum state n=5, k=4, j=-2, s=1 is

$$E_{5f7} = -\frac{D}{25}\frac{34}{50}\left(1 - \frac{9\,\alpha^2}{1250}\right) = -\,0.74159585191 \text{ eV} \qquad (102)$$

The state with quantum numbers n=5, k=4, j=-3, s=3/2 has energy

$$E_{5f8} = -\frac{D}{25}\frac{34}{50}\left(1 - \frac{81\,\alpha^2}{5000}\right) = -\,0.74159549638 \text{ eV} \qquad (103)$$

The quantum state n=5, k=4, j=-4, s=2 has energy

$$E_{5f9} = -\frac{D}{25}\frac{34}{50}\left(1 - \frac{36\,\alpha^2}{1250}\right) = -\,0.7415949988 \text{ eV} \qquad (104)$$

The state with quantum numbers n=5, k=4, j=-5, s=5/2 has energy

$$E_{5f10} = -\frac{D}{25}\frac{34}{50}\left(1 - \frac{225\,\alpha^2}{5000}\right) = -\,0.74159435917 \text{ eV} \qquad (105)$$

1.5.5 k=5 , sub-level 5q

The quantum state n=5, k=5, j=1, s=1/2 has energy

$$E_{5q1} = -\frac{D}{50}\left(1 - \frac{\alpha^2}{5000}\right) = -\,0.54529127087 \text{ eV} \qquad (106)$$

The quantum state n=5, k=5, j=2, s=1 has energy

$$E_{5q2} = -\frac{D}{50}\left(1 - \frac{\alpha^2}{1250}\right) = -\,0.54529125323 \text{ eV} \qquad (107)$$

Energy of the quantum state n=5, k=5, j=3, s=3/2 is given by

$$E_{5q3} = -\frac{D}{50}\left(1 - \frac{9\,\alpha^2}{5000}\right) = -\,0.5429122433 \text{ eV} \qquad (108)$$

Energy of the quantum state n=5, k=5, j=4, s=2 is given by

$$E_{5q4} = -\frac{D}{50}\left(1 - \frac{2\,\alpha^2}{625}\right) = -\,0.5452911837 \text{ eV} \qquad (109)$$

Energy of the quantum state n=5, k=5, j=5, s=5/2 is given by

$$E_{5q5} = -\frac{D}{50}\left(1 - \frac{25\,\alpha^2}{5000}\right) = -\,0.54529113135\ \text{eV} \qquad (110)$$

Energy of the quantum state n=5, k=5, j=-1, s=1/2 is given by

$$E_{5q6} = -\frac{D}{50}\left(1 - \frac{9\,\alpha^2}{5000}\right) = -\,0.54529122433\ \text{eV} \qquad (111)$$

The state with quantum numbers n=5, k=5, j=-2, s=1 has energy

$$E_{5q7} = -\frac{D}{50}\left(1 - \frac{9\,\alpha^2}{1250}\right) = -\,0.54529106755\ \text{eV} \qquad (112)$$

The state with quantum numbers n=5, k=5, j=-3, s=3/2 has energy

$$E_{5q8} = -\frac{D}{50}\left(1 - \frac{81\,\alpha^2}{5000}\right) = -\,0.54529080609\ \text{eV} \qquad (113)$$

The state with quantum numbers n=5, k=5, j=-4, s=2 has energy

$$E_{5q9} = -\frac{D}{50}\left(1 - \frac{36\,\alpha^2}{1250}\right) = -\,0.5452904402\ \text{eV} \qquad (114)$$

The quantum state n=5, k=5, j=-5, s=5/2 has energy

$$E_{5q10} = -\frac{D}{50}\left(1 - \frac{225\,\alpha^2}{5000}\right) = -\,0.54528996988\ \text{eV} \qquad (115)$$

2. Considerations on energy levels of hydrogen atom

Calculations of preceding paragraphs show there are numerous states with different quantum numbers that have the same value of energy. It doesn't represent a strangeness because every quantum state is characterized by a different set of four quantum numbers and it is possible that two states with different quantum numbers can have the same value of energy. It happens for instance for energy levels E_{3s4} and E_{3s5} , for E_{3p3} and E_{3p4} , for E_{3d3} and E_{3d4} , for E_{4s3} and E_{4s5} , for E_{4p3} and E_{4p5} , for E_{4d3} and E_{4d5} , etc... . Besides considering levels of energy are negative and they increase, in relative value, when the quantum number increases, it is possible to observe (fig.1)

1. The sub-level 3d is more external than the sub-level 4s
2. The sub-level 4f is more external than sub-levels 5s, 5p, 5d

Considering values of energy in general increase, in negative value, when the quantum number n of level increases, it is possible that sub-levels of a greater quantum level can have smaller values of energy than sub-levels of a smaller quantum level.
At last in order that the electron in the fundamental state 1s1 can jump to the state 5q10, it needs it absorbs a photon with energy $\Delta E = E_{5q10} - E_{1s1} = 13.08$ eV = $20.93 \cdot 10^{-19}$J. This photon has frequency $f = \Delta E/h = 3.157 \cdot 10^{15}$ Hz and it is in the ultraviolet band.

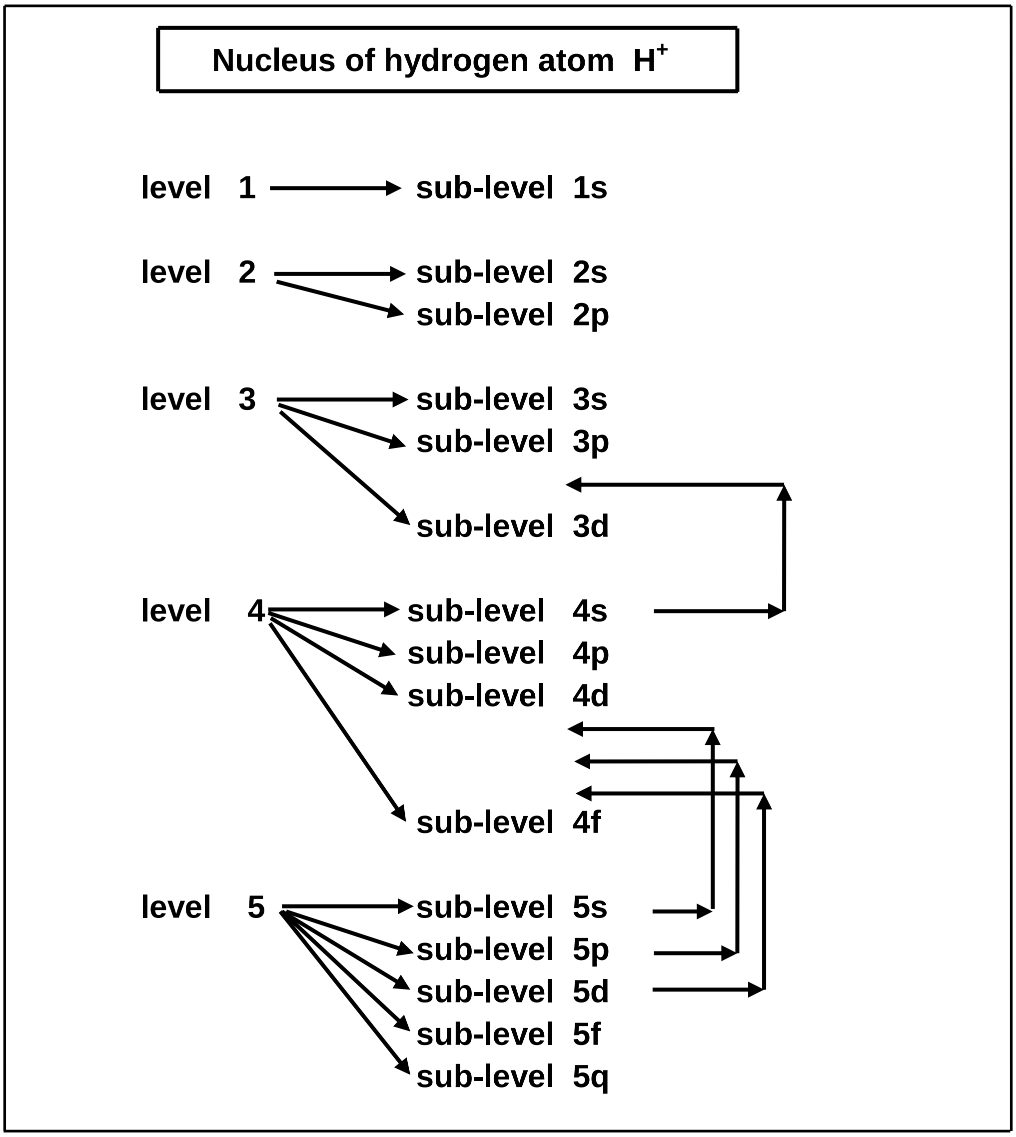

Fig.1 Graphic representation of energy levels and sub-levels of hydrogen atom

References

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