The title

Proof to the twin prime conjecture

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Abstract

The elementary proof to the twin prime conjecture.

The content of the article

Let p_s denote the s'th prime and P_s the product of the first s primes.

Define A_s to be the set of all positive integers less than P_s which are relatively prime to P_s.

1. Each A_s, for $s \geq 3$, contains two elements which differ by 2.

2. Consider the finite arithmetic progression $\{a + mP_s\}$, where a is in A_s and $0 \leq m < P_s$. More than half of the elements are prime.

3. Combining 1) and 2), there is always a pair of twin primes which are relatively prime to P_s, and therefore infinitely many twin primes.

For every pair of values a, b in A_s differing by d, there exist at least $p_{s+1} - 2$ pairs of values in A_{s+1} differing by d. (And exactly that many when d is not divisible by p_{s+1}).

Given this, the claim follows using induction with $d = 2$, noting for the base case that $11, 13$ are both in A_3.

The proof is as follows: Suppose a and b are in A_s, with $b - a = d$. Consider the set of values $a + mP_s$, where $0 \leq m < p_{s+1}$. These are all less than P_{s+1}, and since P_s is relatively prime to p_{s+1}, there is a unique value $m1$ with $a + m1P_s$ divisible by p_{s+1}. Similarly, there is a unique value $m2$ with $b + m2P_s$ divisible by p_{s+1}. Furthermore, if $m1 = m2$, then $(b + m2P_s) - (a + m1P_s) = d$ would be divisible by p_{s+1}. So when d is not divisable by p_{s+1}, for the $p_{s+1} - 2$ values of $0 \leq m < p_{s+1}$ which are not equal to $m1$ or $m2$, the pair $(a + mP_s, b + mP_s)$ are a pair in A_{s+1} differing by d.

Proof of 2

Consider the finite arithmetic progression $\{a + mP_s\}$, where a is in A_s and $0 \leq m < P_s$. More than half of the elements are prime.

The largest number generate by $a + mP_s = {P_s}^2 - 1$ is when $a = P_s - 1$ and $m = P_s - 1$

Able to approximate all the non-prime numbers generated by the arithmetic progression $P_s - 1 + mP_s$ where $0 \leq m < P_s$ with arithmetic progression $0 + n(P_s - 1)$ where $1 \leq n \leq P_s + 1$.

The first and last terms of the two arithmetic progressions are equal: $P_s - 1 + 0 \times P_s = 0 + 1 \times (P_s - 1)$ and $P_s - 1 + (P_s - 1)P_s = 0 + (P_s + 1)(P_s - 1)$.

And the approximate arithmetic progression common difference is smaller: $P_s - 1 < P_s$.

Therefore able to generate non-overlapping P_s integer intervals between and inclusive of P_s-1 to $(P_s-1)(P_s+1)$ such that each interval only contains a number from P_s-1+mP_s.

The inclusive intervals are as follows:

P_s-1 to $2\times(P_s-1)-1$

$2\times(P_s-1)$ to $3\times(P_s-1)-1$

$3\times(P_s-1)$ to $4\times(P_s-1)-1$

$4\times(P_s-1)$ to $5\times(P_s-1)-1$

...

$(\frac{P_s}{2}+1)\times(P_s-1)$ to $\frac{P_s+2}{2}\times(P_s-1)-1$

$\frac{P_s+2}{2}\times(P_s-1)$ to $(\frac{P_s+2}{2}+1)\times(P_s-1)$

$(\frac{P_s+2}{2}+1)\times(P_s-1)+1$ to $(\frac{P_s+2}{2}+2)\times(P_s-1)$

...

$(P_s+1-3)\times(P_s-1)+1$ to $(P_s+1-2)\times(P_s-1)$

$(P_s+1-2)\times(P_s-1)+1$ to $(P_s+1-1)\times(P_s-1)$

$(P_s+1-1)\times(P_s-1)+1$ to $(P_s-1)\times(P_s+1)$

All terms of $0+n(P_s-1)$ when $n>1$ are non-prime numbers.

Assume $1\times(P_s-1)$ is non-prime.

Apply the restriction that all non-prime numbers must be odd to arithmetic progression $0+n(P_s-1)$

There are already least $(P_s+1)-\frac{P_s+2}{2}$ prime numbers in arithmetic progression P_s-1+mP_s after converting $0+n(P_s-1)$ into P_s-1+mP_s.

The first and last terms of the two arithmetic progressions are equal: $P_s-1+0\times P_s=0+1\times(P_s-1)$ and $P_s-1+(P_s-1)P_s=0+(P_s+1)(P_s-1)$

To convert form arithmetic progression $0+n(P_s-1)$ to arithmetic progression P_s-1+mP_s where $1\le n\le P_s+1$ and $0\le m<P_s$.

First remove the term $0+\frac{P_s+1+1}{2}\times(P_s-1)$.

For all terms less than $0+\frac{P_s+1+1}{2}\times(P_s-1)$ and greater than $0+1\times(P_s-1)$ add a positive integer.

For all terms greater than $0+\frac{P_s+1+1}{2}\times(P_s-1)$ and less than $0+(P_s+1)\times(P_s-1)$ substract a positive integer.

Therefore revising the number of prime numbers in arithmetic progression P_s-1+mP_s to be at least $(P_s)-(\frac{P_s+2}{2}-2)$

$$\frac{P_s+1+1}{2}-2<\frac{P_s}{2}$$

Now consider the finite arithmetic progression $\{a+mP_s\}$, where a is in A_s and $0\le m<P_s$ and $a\ne P_s-1$.

The approximate arithmetic progress $0+n(P_s-1)$ can be adjusted to become $-(P_s-1)+a+n(P_s-1)$ where $1\le n\le P_s+1$.

For example $1 + mP_s$ has the approximate arithmetic progression $-(P_s - 1) + 1 + n(P_s - 1)$.

The value of a is relatively prime to $P_s - 1$. Therefore for all odd numbers in approximate arithmetic progression $-(P_s-1)+a+n(P_s-1)$ where $3 \leq n \leq P_s - 1$ to be odd non-prime numbers then for each $-(P_s - 1) + a + n(P_s - 1)$ must be divisable by n. But $-(P_s - 1) + a + 3(P_s - 1)$ is not divisable by 3. But $-(P_s - 1) + a + (P_s - 1)(P_s - 1)$ is not divisable by $(P_s - 1)$. Therefore there are at least 2 odd numbers in $-(P_s - 1) + a + n(P_s - 1)$ which are prime.

Therefore the maximum number of possible non-prime generated which could be non-prime numbers in actual arithmetic progression is $\frac{P_s+1+1}{2} - 2$

$$\frac{P_s + 1 + 1}{2} - 2 < \frac{P_s}{2}$$

`https://www.reddit.com/r/badmathematics/comments/aljw4b /elementary_proof_to_the_twin_prime_conjecture_to/`

`User Leet_Noob rewrote proof structure and proof to 1.`