The title

Proof to the twin prime conjecture

Abstract

The elementary proof to the twin prime conjecture.

The content of the article Let p_s denote the s'th prime and $P_s$the product of the first s primes.

Define A_s to be the set of all positive integers less than P_s which are relatively prime to P_s

1. Each A_s, for $s \geq 3$, contains two elements which differ by 2.

2. Consider the finite arithmetic progression $\{a + mP_s\}$, where a is in A_s and $0 \leq m < P_s$. More than half of the elements are prime.

3. Combining 1) and 2), there is always a pair of twin primes which are relatively prime to P_s, and therefore infinitely many twin primes.

For every pair of values a, b in A_s differing by d, there exist at least $p_{s+1} - 2$ pairs of values in A_{s+1} differing by d. (And exactly that many when d is not divisible by p_{s+1}).

Given this, the claim follows using induction with $d = 2$, noting for the base case that $11, 13$ are both in A_3.

The proof to 1 is as follows: Suppose a and b are in A_s, with $b - a = d$. Consider the set of values $a + mP_s$,

where $0 \leq m < p_{s+1}$. These are all less than P_{s+1}, and since P_s is relatively prime to p_{s+1},

there is a unique value $m1$ with $a + m1P_s$ divisible by p_{s+1}.

Similarly, there is a unique value $m2$ with $b + m2P_s$ divisible by p_{s+1}.

Furthermore, if $m1 = m2$, then $(b + m2P_s) - (a + m1P_s) = d$ would be divisible by p_{s+1}.

So when d is not divisible by p_{s+1}, for the $p_{s+1} - 2$ values of $0 \leq m < p_{s+1}$ which are not equal to $m1$ or $m2$, the pair $(a + mP_s, b + mP_s)$ are a pair in A_{s+1} differing by d.

The proof to 2 is as follows:

Consider the finite arithmetic progression $\{a + mP_s\}$, where a is in A_s and $0 \leq m < P_s$. More than half of the elements are prime.

Suppose $a = P_s - 1$

Able to approximate all the non-prime numbers generated by the arithmetic progression $P_s - 1 + mP_s$ where $0 \leq m < P_s$ with arithmetic progression $0 + n(P_s - 1)$ where $1 \leq n \leq P_s + 1$.

Note that the first and last terms of the two arithmetic progressions are equal: $P_s - 1 + 0 \times P_s = 0 + 1 \times (P_s - 1)$ and $P_s - 1 + (P_s - 1)P_s = 0 + (P_s + 1)(P_s - 1)$.

All terms of $0 + n(P_s - 1)$ when $n > 1$ are non-prime numbers.

Assume $0 + 1 \times (P_s - 1)$ is non-prime.

Apply the restriction that all non-prime numbers must be odd in arithmetic progression $P_s - 1 + mP_s$ to arithmetic progression $0 + n(P_s - 1)$

There are $\frac{P_s+2}{2}$ odd numbers out of $P_s + 1$ terms in $0 + n(P_s - 1)$

Therefore there are already naively at least $(P_s) - \frac{P_s+2}{2}$ prime numbers in arithmetic progression $P_s - 1 + mP_s$ after converting terms of $0 + n(P_s - 1)$ into terms of $P_s - 1 + mP_s$.

To convert form arithmetic progression $0 + n(P_s - 1)$ to arithmetic progression $P_s - 1 + mP_s$ where $1 \leq n \leq P_s + 1$ and $0 \leq m < P_s$.

The first and last terms of the two arithmetic progressions are equal: $P_s - 1 + 0 \times P_s = 0 + 1 \times (P_s - 1)$ and $P_s - 1 + (P_s - 1)P_s = 0 + (P_s + 1)(P_s - 1)$.

Reduce the number of terms from $P_s + 1$ to P_s by removing the middle term $0 + \frac{P_s+1+1}{2} \times (P_s - 1)$.

All other terms are converted as such.

$o(P_s - 1) + (o - 1) = P_s - 1 + (o - 1)(P_s)$ where $1 \leq o < \frac{P_s+1+1}{2}$

Not all odd values of $o(P_s - 1)$ where $1 \leq o < \frac{P_s+1+1}{2}$ can be non-prime in $P_s - 1 + mP_s$

$o(P_s - 1) + (P_s - (o - 1)) = P_s - 1 + (o - 1)(P_s)$ where $\frac{P_s+1+1}{2} < o \leq P_s + 1$

Not all odd values of $o(P_s - 1)$ where $\frac{P_s+1+1}{2} < o \leq P_s + 1$ can be non-prime in $P_s - 1 + mP_s$

Therefore revising the number of prime numbers in arithmetic progression $P_s - 1 + mP_s$ to be at least $(P_s) - (\frac{P_s+2}{2} - 2)$

Suppose $a \neq P_s - 1$

The approximate arithmetic progression to $a + mP_s$ is $-(P_s - 1) + a + n(P_s - 1)$ where $1 \leq n \leq P_s + 1$ and $0 \leq m < P_s$.

Again apply the restriction that all non-prime numbers must be odd in $a + mP_s$.

Not all odd values of $-(P_s - 1) + a + o(P_s - 1)$ where $1 \leq o < \frac{P_s+1+1}{2}$ can be non-prime in $a + mP_s$

Not all odd values of $-(P_s - 1) + a + o(P_s - 1)$ where $\frac{P_s+1+1}{2} < o \leq P_s + 1$ can be non-prime in $a + mP_s$

Therefore the number of prime numbers in arithmetic progression $a + mP_s$ to be at least $P_s - (\frac{P_s+2}{2} - 2)$