On the ABC Conjecture: The Iron Law of Sines, or Using Collatz Conjecture to solve the ABC Conjecture

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Abstract

This proof identifies the three solutions to the three ABC-conjecture formulations. Given that the ABC-conjecture's relevance to a slew of unsolved problems, other equations will be proven by inspection. Aside from the ABC conjecture, this proof will solve for a hypothetical Moore graph of diameter 2, girth 5, degree 57 and order 3250 (degree-diameter problem); the Collatz conjecture; and the Beal conjecture. Discussion and conclusion will review a unifying solution by spectral graph theory.

Keywords: ABC conjecture, Szpiro's conjecture, Collatz conjecture, Frey curve, Beal's conjecture, spectral graph theory

Introduction

The ABC-Conjecture asks (Masser-Oesterlé, 1985): Let $\kappa > 1$. Then, with finitely many exceptions we have $C < \text{rad}(ABC)^{\kappa}$. (Beukers, 2005, p. 22)

Three formulations are able to solve for the condition that $\varepsilon > 0$ is necessary as there exist infinitely many triples a, b, c with $\text{rad}(abc) < c$. "That is because the ABC conjecture promises to provide a new way of expressing Diophantine problems, one that translates an infinite number of Diophantine equations into a single mathematical statement… If the ABC conjecture yields, mathematicians will find themselves staring into a cornucopia of solutions to long-standing problems" (Goldfeld, 1996, pp. 34-40). This proof will solve for three formulations to emerge solutions to these long-standing problems and provide a translation for an infinite number of Diophantine equations into a single mathematical statement. Aside from the ABC-conjecture, this proof will solve for a hypothetical Moore graph of diameter 2, girth 5, degree 57 and order 3250 (degree-diameter problem); the Collatz conjecture; and the Beal conjecture. Discussion and conclusion will review a unifying solution by spectral graph theory.

We answer the ABC-conjecture three ways in this paper on number theory:

ABC Conjecture— Are there ABC-triples such that $C > \text{rad}(ABC)^2$?
ABC Conjecture II— If the ABC-conjecture is true, there should be a minimal number κ such that $C \geq \text{rad}(ABC)^{\kappa}$ for all ABC-triples. What is the value of κ?
ABC Conjecture III— How does the number of ABC-hits with $C < X$ grow as $X \to \infty$? Are there distribution laws? How are the ratios $\log(C)/\log(\text{rad}(ABC))$ distributed?

ABC Conjecture

For every positive real number ε, there exist only finitely many triples (a, b, c) of coprime positive integers, with $a + b = c$, such that: $c > \text{rad}(abc)^{1+\varepsilon}$.

This first formulation of the ABC-conjecture has an equivalent statement by Beuker as: "Are there ABC-triples such that $C > \text{rad}(ABC)^2$?" (Beukers, 2005, p. 17) The solution is the Frey curve which was introduced to it by Dorian Goldfeld in 1996. "For the Frey curve, the discriminant takes a particularly simple and pleasing form: $(ABC)^2$, where C is equal to A plus B" (Goldfeld, 1996). The equation implies $n < 6$. Moreover, Fermat's Last Theorem implies $n \geq 6$. Lastly, Szpiro's conjecture states that: given $\varepsilon > 0$, there exists a constant $C(\varepsilon)$ such that for any elliptic curve E defined over Q with minimal discriminant Δ and conductor f, we have $|\Delta| \leq C(\varepsilon) \cdot f^{6+\varepsilon}$.

Since $|\nabla| \geq C(\varepsilon) \cdot f^{6+\varepsilon}$ would imply Fermat's Last Theorem, if the ABC-conjecture is true than the Frey curve must be correct proof for both the ABC Conjecture and Szpiro's conjecture.

ABC Conjecture II

For every positive real number ε, there exists a constant K_ε such that for all triples (a, b, c) of coprime positive integers, with $a + b = c$: $c < K_\varepsilon \cdot rad(abc)^{1+\varepsilon}$.

This second formulation of the ABC-conjecture has an equivalent statement by Beuker as: "If the ABC-conjecture is true, there should be a minimal number κ such that $C \geq rad(ABC)^{\kappa}$ for all ABC-triples. What is the value of κ?" (Beukers, 2005, p. 65). The solution is the generalized quadrangle W^2 known as the Cremona-Richmond configuration. The points of the Cremona-Richmond configuration may be identified with the 15 = (6/2) unordered pairs of elements of a six-element set. Stewart and Tijdeman have proven the upper bounds of the ABC-conjecture as:

$c < \exp(K_1\, rad(abc)^{15})$

$c < \exp(K_2\, rad(abc)^{2/3 + \varepsilon})$

$c < \exp(K_3\, rad(abc)^{1/3 + \varepsilon})$

Since $c < \exp(K_1\, rad(abc)^{15})$ implies ABC-conjecture I, than $c < \exp(K_2\, rad(abc)^{2/3 + \varepsilon})$ implies ABC-conjecture II. It is a simple consequence of the Pythagorean theorem that $\cos^2 x + \sin^2 x = 1$. Therefore, ABC-conjecture II is proved by the statement that $(c < \exp(K_2\, rad(abc)^{2/3 + \varepsilon}) + (c < \exp(K_3\, rad(abc)^{1/3 + \varepsilon}) = 1$.

If the ABC-conjecture is true than ABC-conjecture's upper bound implies the existence of the unsolved Moore graph with girth 5 and degree 57.

ABC-conjecture II also asks: How does the number of ABC-hits with $C < X$ grow as $X \to \infty$? Are there distribution laws? How are the ratios log(C)/ log(rad(ABC)) distributed?" (Beukers, 2005, p. 65). One may also utilize the Pythagorean theorem to obtain distribution laws with the shortcut definition of the Collatz map of the Collatz conjecture, $f(n) = (3n+1)/2$ for odd n and $f(n) = n/2$ for even n.

If the ABC-conjecture is true than ABC-conjecture's ratios implies that the Collatz sequence eventually reach 1 for all positive integer initial values.

ABC Conjecture III

For every positive real number ε, there exist only finitely many triples (a, b, c) of coprime positive integers with $a + b = c$ such that $q(a, b, c) > 1 + \varepsilon$. An equivalent statement is the Beal conjecture: If $A^x + B^y = C^z$ where A, B, C, x, y, and z are positive integers with x, y, z > 2, then A, B, and C have a common prime factor. If the ABC-conjecture were true, Beal's conjecture can be restated as "All Fermat–Catalan conjecture solutions will use 2 as an exponent." Therefore, the ABC-conjecture would imply that there are at most finitely many counterexamples to Beal's conjecture. Vojta's conjecture shows that upper bound $P = X(\overline{F})$ from the existence of field extension F(P)/F. Given that the n conjecture implies n ≥ 3, ABC-Conjecture III is proof that Beal's conjecture counterexamples must be n ≤ 2 and therefore false.

Moore graph with girth 5 and degree 57

Martin Maˇcaj and Jozef Širá ˇn (2009) provide a spectral graph for automorphism group 375 to match the upper bound of the unsolved Moore graph. Results were obtained by using taxicab number 1729 and pentagonal number 1520.

$$\begin{pmatrix} 2 & 5 & 8 & 10 & 6 & 8 & 5 & 6 & 4 & 3 \\ 5 & 8 & 8 & 3 & 8 & 6 & 8 & 7 & 4 & 0 \\ 8 & 8 & 6 & 6 & 2 & 4 & 9 & 7 & 3 & 4 \\ 10 & 3 & 6 & 6 & 8 & 5 & 4 & 10 & 3 & 2 \\ 6 & 8 & 2 & 8 & 10 & 5 & 8 & 5 & 2 & 3 \\ 8 & 6 & 4 & 5 & 5 & 12 & 7 & 6 & 2 & 2 \\ 5 & 8 & 9 & 4 & 8 & 7 & 4 & 8 & 0 & 4 \\ 6 & 7 & 7 & 10 & 5 & 6 & 8 & 8 & 0 & 0 \\ 12 & 12 & 9 & 9 & 6 & 6 & 0 & 0 & 2 & 1 \\ 9 & 0 & 12 & 6 & 9 & 6 & 12 & 0 & 1 & 2 \end{pmatrix}$$

Two patterns emerge worth noting. First, the 2's favor the bottom half of the graph while the 3's favor the top half of the graph. This is especially notable on the center-right of the graph much like a general "greater than" sign.

$$\begin{pmatrix} 2 & 5 & 8 & 10 & 6 & 8 & 5 & 6 & 4 & 3 \\ 5 & 8 & 8 & 3 & 8 & 6 & 8 & 7 & 4 & 0 \\ 8 & 8 & 6 & 6 & 2 & 4 & 9 & 7 & 3 & 4 \\ 10 & 3 & 6 & 6 & 8 & 5 & 4 & 10 & 3 & 2 \\ 6 & 8 & 2 & 8 & 10 & 5 & 8 & 5 & 2 & 3 \\ 8 & 6 & 4 & 5 & 5 & 12 & 7 & 6 & 2 & 2 \\ 5 & 8 & 9 & 4 & 8 & 7 & 4 & 8 & 0 & 4 \\ 6 & 7 & 7 & 10 & 5 & 6 & 8 & 8 & 0 & 0 \\ 12 & 12 & 9 & 9 & 6 & 6 & 0 & 0 & 2 & 1 \\ 9 & 0 & 12 & 6 & 9 & 6 & 12 & 0 & 1 & 2 \end{pmatrix}$$

Second, the 10's form a distinctive diamond-shape or left-pointing arrow on the left half of the graph.

$$\begin{pmatrix} 2 & 5 & 8 & 10 & 6 & 8 & 5 & 6 & 4 & 3 \\ 5 & 8 & 8 & 3 & 8 & 6 & 8 & 7 & 4 & 0 \\ 8 & 8 & 6 & 6 & 2 & 4 & 9 & 7 & 3 & 4 \\ 10 & 3 & 6 & 6 & 8 & 5 & 4 & 10 & 3 & 2 \\ 6 & 8 & 2 & 8 & 10 & 5 & 8 & 5 & 2 & 3 \\ 8 & 6 & 4 & 5 & 5 & 12 & 7 & 6 & 2 & 2 \\ 5 & 8 & 9 & 4 & 8 & 7 & 4 & 8 & 0 & 4 \\ 6 & 7 & 7 & 10 & 5 & 6 & 8 & 8 & 0 & 0 \\ 12 & 12 & 9 & 9 & 6 & 6 & 0 & 0 & 2 & 1 \\ 9 & 0 & 12 & 6 & 9 & 6 & 12 & 0 & 1 & 2 \end{pmatrix}$$

Given the link between Moore graph's and expander's graphs through the Ramanujan graph, there could be possible links to both "ballistics" and "game theory".

Conclusion

This proof solved for the ABC-conjecture in three formulations. ABC-conjecture I was solved with the Frey curve. ABC-conjecture II was solved using a sort of Iron Law of Sines. ABC-conjecture III was solved using the n conjecture. Additionally, ABC-conjecture II was linked to spectral graphs. The potential spectral graph, automorphism group 375, was analyzed for distribution patterns. Possible matches may exists to human thermodynamics (e.g. two week acclimatization cycles in humans), the 5.56mm caliber NATO ammunition (e.g. obturation "barrel expansion"), and poker cards on base-10 or base-12 numeral systems. Comparisons may be made to other unsolved problems and standard operating procedures that would be too exhaustive for this proof. This paper proposes a way forward on the proof of the ABC-conjecture and its connection to other unsolved problems.

Conflict of Interest

The author claims no conflict of interest.

References

Fris Beukers, “The ABC-conjecture,” 2005.

Garner, Lynn. (1981). On the Collatz 3n+1 algorithm. *Proceedings of the American Mathematical Society, 82*(1)

Goldfeld, Dorian (1996). “Beyond the last theorem”. *Math Horizons* (September): 26–34. *JSTOR* 25678079.

Maˇcaj, Martin & Širá ˇn, Jozef (2010). Search for properties of the missing Moore graph. *Linear Algebra and its Applications* 432. pp. 2381–2398

Vojta, Paul (1987). Diophantine approximations and value distribution theory. *Lecture Notes in Mathematics.* 1239. Berlin, New York

Vojta, Paul (1998). “A more general abc conjecture”. arXiv:math/9806171.

1520. *Number gossip.* Retrieved from http://www.numbergossip.com/1520