Denial of schematic refutations of formula schemata

Abstract: We evaluate a definition of the schematic formula of the proposed framework. It is *not* tautologous and hence denies these particular refutations of formula schema. The example of the pigeon hole principle, a trivial theorem, is also not refuted by the proposed framework.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪; - Not Or; & And, ∧, ∩, · ; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ⊢, ⊨, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲;
= Equivalent, ≡, :=, ⟺, ↔, ≜, ≈, ≃; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;
(%z<#z) `C` as contingency, Δ, ordinal 1; (%z>#z) `N` as non-contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

From: Cerna, D.; Leitsch, A.; Lolic, A. (2019). Schematic refutations of formula schemata.
arxiv.org/pdf/1902.08055.pdf
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Abstract: Proof schemata are infinite sequences of proofs which are defined inductively. In this paper we present a general framework for schemata of terms, formulas and unifiers and define a resolution calculus for schemata of quantifier-free formulas. The new calculus generalizes and improves former approaches to schematic deduction.

As an application of the method we present a schematic refutation formalizing a proof of a weak form of the pigeon hole principle. (0.0)

Remark 0.0: The text does not directly describe the pigeon hole principle, but cites a reference, so we invoke en.wikipedia.org/wiki/Pigeonhole_principle.

If n objects are distributed over m places, and if n < m, then some place receives no object. (1.0)

Remark 1.0: The mechanism of distributing n over m is not exactly explained, and the word "some" is not defined. Therefore we write Eq. 1.0 by replacing "place" with "space" to mean:

If n objects are less than m spaces and some object implies the necessity of space, then some object implies the possibility of no space. (1.1)

LET p, q: m spaces, n objects.

((q<p)&(%q>#p))>(%q>%~p) ; TTTT TTTT TTTT TTTT (1.2)

Remark 1.2: As rendered in Eq. 1.2, the pigeon hole principle is tautologous and a trivial theorem. It is the stronger form of the theorem. The weaker form, to which the paper directs, in this context substitutes the antecedent clause of "some object implies the necessity of space" with "some object implies the *possibility* of space", for result of the same table.

Definition 12 (formula schemata (FS)). We define the set FS inductively:

– Let F ∈FS then ¬F ∈FS. (12.4.1)

LET p, q, r, s: F, F_1, F_2, S.

(p<(p&s))>(~p<(p&s)) ; T**F**T**F** T**F**T**F** TTTT TTTT (12.4.2)

Remark 12.5.2: Eq. 1.5.2 is *not* tautologous, hence denying these particular schemtic refutations of formula schemata.