

On the Equivalence of Closed Figures and Infinitely extended Lines and the Conclusions drawn from it

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Abstract

The equivalence of closed figures and infinitely extended lines may lead us to understand the physical reality of infinities. This paper doesn't include what infinities mean in the physical world, but the paper is mainly focused on the equivalence of closed figures and infinitely extended lines. Using this principle, some major conclusions can be drawn. The equivalence of closed figures and infinitely extended lines is mainly based on the idea that closed figures and infinitely extended lines are equivalent. One of the most significant conclusions drawn from this equivalency is that if any object moves along a straight infinitely extended line, it will return back to the point, where it started to move, after some definite time.

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1. INTRODUCTION

Infinitely extended lines are one of the three types of set of (continuous) collinear points. As the points are continuous, they have no distance between them; so, it gives rise to a line (infinitely extended in both directions); a ray (infinitely extended in one direction) or a line segment (having a finite length). All the three are extended in only one dimension of space. The term '*infinitely extended lines*' is used for the first type of set of collinear points, that is, line.

A line is extended infinitely in both directions. It neither has a definite starting point nor ending point. A ray is extended infinitely only in a single direction. It has a definite starting point but not an ending point (or vice versa). A line segment is a finite set of (continuous) collinear points. It has a definite starting point as well as a definite ending point.

The similarity between lines and rays is that both of them have infinite length (as they are extended infinitely in single dimension of space) but a line segment has a definite and measurable length. The similarity in all the three is that they are single dimensional.

Closed figures are the shapes which are closed. Here closed figures refer to the shapes which are closed and extended in two dimensions of space. Two-dimensional closed figures can be broadly classified into two categories, curved closed figures and polygons.

Curved closed figures can be thought of as a line segment which is curled up in such a way that its starting and ending points coincide each other, giving a closed figure (Figure 1).

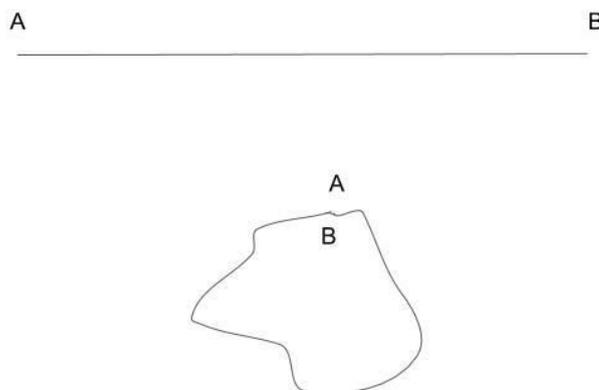


Figure 1 - A perfectly one dimensional line segment (AB), curled up to form a curved closed figure in such a way that the points A and B coincide.

A polygon is a closed figure which is made up of line segments. The line segments are joined end to end, in such a way that a closed figure is obtained. A polygon has minimum of three sides, because if two line segments are joined end to end, a further longer line segment is obtained.

2. TWO-DIMENSIONAL CLOSED FIGURES

Two-dimensional closed figures are the shapes which are closed and are extended in two dimensions (of space). Two-dimensional closed figures can be broadly classified into two categories, curved closed figures and polygons.

Curved closed figures can be thought of as a line segment which is curled up in such a way that its starting and ending points coincide each other, giving a closed figure (Figure 1).

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2.1 POLYGONS

Polygon is a closed figure, extended in two dimensions (of space) and made up of (finite and measurable) line segments. The word 'polygon' is derived from two Greek words '*poly*' means 'many' or 'much' and '*gon*' means corners or angles [1].

The line segments are joined end to end, in such a way that a closed figure is obtained. A polygon has minimum of three sides, because if two line segments are joined end to end, a further longer line segment is obtained.

As a polygon is made up of line segments of finite length, it also has a finite perimeter (length of boundary) and takes up a finite area in two-dimensional space.

Polygons, on the basis of the number of sides, can be classified into a number of shapes. The number of sides, n , of a polygon has the minimum value of *three* and can go up to *infinity*. A polygon having $n = 3$, is called a *3-gon* or triangle. Similarly polygon having n

= 4, 5, 6, 7, 8, 9 or 10 is called a quadrilateral, pentagon, hexagon, heptagon, octagon, nonagon or decagon, or, *4-gon*, *5-gon*, *6-gon*, *7-gon*, *8-gon*, *9-gon* or *10-gon* respectively (a polygon with n number of sides is called n -gon). Figure 2 shows the polygon upto *12-gon* or dodecagon.

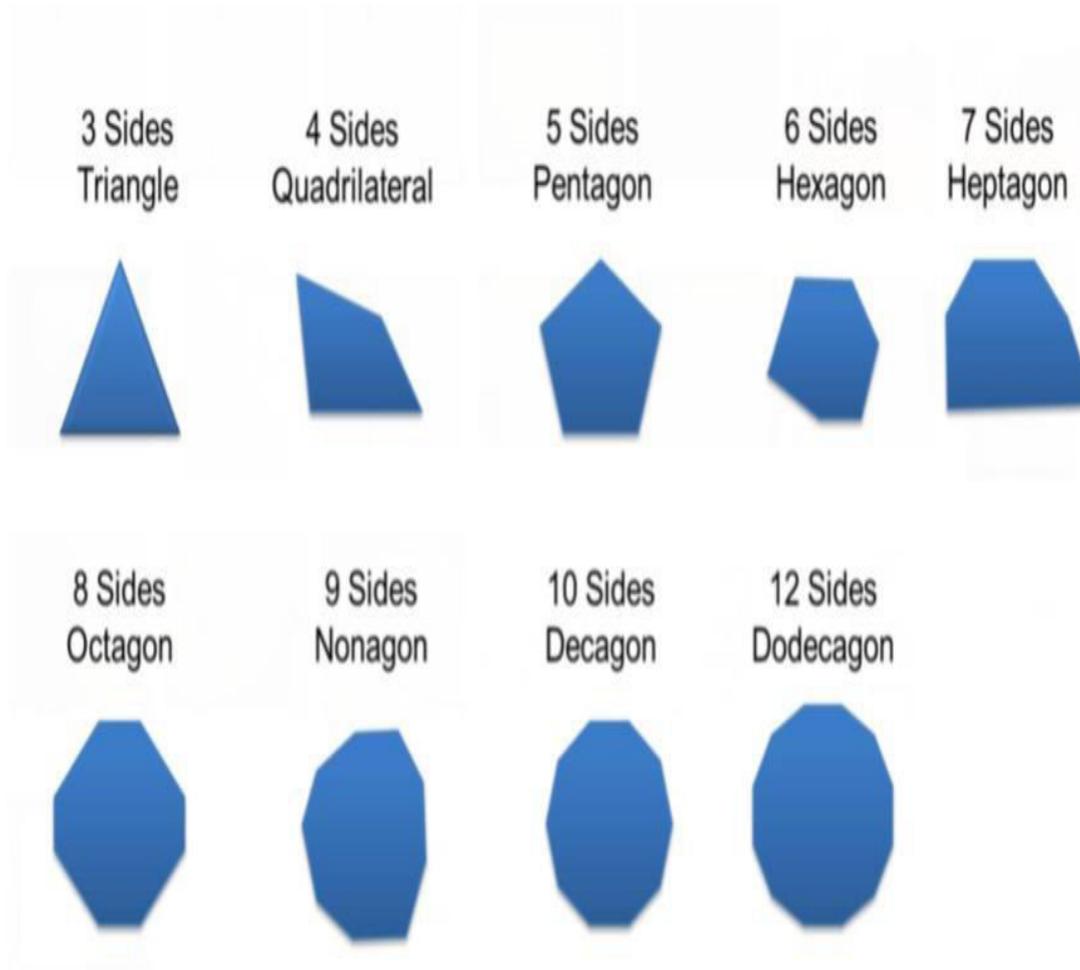


Figure 2 - Polygons up to number of sides, $n = 12$.

2.1.1 CIRCLE AS A POLYGON?

It can be observed from Figure 2, that, as the number of sides increases, the polygon comes near to a 'circle-like' figure, that is, it starts to look somewhat like a circle but made up of line segments. So, the idea is that as numbers of sides become infinite, a polygon will become a perfect circle. But, this is not the case in the physical reality we are talking about. Something made up of infinite line segments must have an infinite length and area.

So, speaking about physical reality, a circle should be considered as a perfect single dimensional line segment curled up in such a way that the starting and ending point of the line segment coincide and any point taken on this curled line segment is equidistant from a particular point inside the curled line segment, which is the center of the circle (Figure 3). Now, circle has a finite length (perimeter) and area.

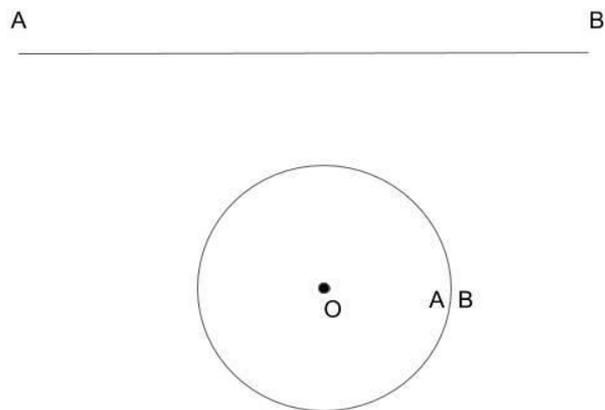


Figure 3 - A perfect single dimensional line segment, AB, curled up in such a way that the points A and B of the line segment coincide and any point taken on this curled line segment is equidistant from a particular point inside the curled line segment, O, which is said to be the center of the circle.

2.1.2 APEIROGON

A polygon having infinite number of sides is not a circle, but, is an *apeirogon*. An apeirogon is a polygon with number of sides, $n = \text{infinite}$ [2]. The interior of a linear apeirogon can be defined by a counterclockwise orientation of vertices, as shown by arrows on the edges in Figure 4, defining the top half plane in Figure 4 [3 - 4].

Interior



Figure 4 - The interior of a linear apeirogon can be defined by a counterclockwise orientation of vertices, as shown by arrows on the edges in the figure, defining the top half plane in the figure.

3. PRINCIPLE OF EQUIVALENCE OF CLOSED FIGURES AND INFINITELY EXTENDED LINES

A line is infinitely extended in a single dimension and has infinite length, and, closed figures are extended in two dimensions and have finite size. But, the similarity between closed figures and infinitely extended lines is that neither a starting point nor an ending point can be given to closed figures and infinitely extended lines.

Suppose an object starts moving (in respect to an observer at rest) from a point 'A' on a closed figure. It moves along the boundary of the closed figure. Let's say that the object marks the point 'A' as the starting point of the closed figure, but still it will be unable to give a definite ending point to the closed figure.

Now, suppose, the object starts moving (in respect to an observer at rest) from a point 'B' on an infinitely extended line. It moves along the line. Let's say the object marks 'B' as the starting point of the infinitely extended line, but still it will be unable to give a definite ending point to the infinitely extended line.

This leads to the idea that closed figures and infinitely extended lines are equivalent, that is, they are similar but not identical. This is the main principle.

3.1 CONCLUSIONS DRAWN FROM THE EQUIVALENCE OF CLOSED FIGURES AND INFINITELY EXTENDED LINES

With help of the current version of the principle, two main ‘phenomena’ are derived which are as follows:

1. If an object starts moving from a point ‘A’ on a closed figure and moves along its boundary, after some definite time, it will return to the same point ‘A’. According to the principle, the same case is with infinitely extended lines. If an object starts moving from a point ‘B’ on an infinitely extended line, it will return to the same point ‘B’ after some definite time.
2. When an object moves along boundary of a closed figure, its direction changes after some definite intervals, so an object moving along an infinitely extended line should also observe acceleration in his motion. This acceleration can be observed in the form of a ‘pseudo force’ as described in Einstein’s Principle of Equivalence that being in an uniformly accelerated frame is equivalent to be in a uniform gravitational field [5].

4. DISCUSSION AND CONCLUSIONS

The principle of equivalence of closed figures and infinitely extended lines as described in this paper, can lead us to understand what infinity means in the physical universe. Actually, in physical reality anything can’t occur in infinite amount, infinities are mathematical, but, we still often see infinities in the physical universe, and, the most common examples are infinite density of singularities [6], infinite gravitational time dilation at event horizon of a black hole as described by the general theory of relativity [7].

The principle can be used as a foundation to understand physical infinities. If we will get some knowledge of what infinities mean in physical universe, then, the general theory of relativity and quantum physics can become consistent with each other [8 - 9] and this principle may have an important role in our understanding of infinity.

Keywords

Physical Infinities; Geometry; Polygons; Lines; Closed Shapes

Image Acknowledgements

[Figure 2] “Information about the Project.” *Ahmed & Sultan's Project*, 24 Feb. 2014,
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[Figure 4] “Apeirogon.” *Wikipedia*, Wikimedia Foundation, 23 July 2018,
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All the other Images not mentioned in this section are developed by the author himself.

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