Confirmed refutation of Kripke-Platek (KP) / Constructive Zermelo-Fraenkel set theory (CZF)

Abstract: We evaluate the set induction scheme for Kripke-Platek set theory (KP) and Constructive Zermelo-Fraenkel set theory (CZF). It is *not* tautologous. This confirms the previous refutation.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪; - Not Or; & And, ∧, ∩, · ; \ Not And; > Imply, greater than, →, ⇒, ↦, ≻, ⊃, ⊢, ⊨, ➝ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↚, ≲;
= Equivalent, ≡, :=, ⟺, ↔, ≜, ≈, ≃; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;
(%z<#z) C as contingency, Δ, ordinal 1; (%z>#z) N as non-contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

From: Weaver, N. (2018). Predicative well-ordering.
arxiv.org/pdf/1811.03543.pdf nweaver@math.wustl.edu

11. Kripke-Platek and CZF

Kripke-Platek set theory (KP) and Constructive Zermelo-Fraenkel set theory (CZF) are two set theoretic systems which are also routinely claimed to be predicative. (According to Wikipedia, KP is "roughly the predicative part of ZFC" and CZF has "a fairly direct constructive and predicative justification".)

In fact, both are impredicative for the same reason ID_1 is: yet again, the fallacy involves a confusion between conditions (A) and (B). In both cases the problematic axioms are the set induction scheme, which states,

for any formula P, $(\forall y)([\forall x \in y\ P(x)] \rightarrow P(y)) \rightarrow (\forall y)P(y)$. (11.1)

LET p, q, r: P, x, y

((#q<(#r&(#p&q)))>(#p&r))>(#p&#q) ; **FF**NN **FF**NN **FF**NN **FF**NN (11.2)

Informally, if a predicate holds of a set y whenever it holds of all the elements of y, that predicate must hold of all sets.

The informal justification for this scheme hinges on the premise that the universe of sets is built up in a well-ordered series of stages. One then applies progressivity of P to infer, inductively, that it holds of all sets in the universe. Just as with ID_1, this justification fails because being well-ordered in the sense of condition (A) does not predicatively entail the instances of condition (B) which would be needed to make the induction argument. And also as in that case, there is no option of strengthening the premise to say that the universe of sets is built up in a series of stages which are well-ordered in some stronger way which affirms condition (B). The instances of condition (B) which we would need in order to justify set induction involve all predicates expressible in the language of set theory, but the latter does not have an interpretation until we specify how the universe of sets is to be built up. So this would be circular.

> **Remark 11.2:** Eq. 11.2 as rendered is not tautologous. This confirms the refutation as impredicative for KP and CZF.