Refutation of definable operators on stable set lattices

Abstract: We evaluate the definitions for the modal operators on stable set lattices. The operators are not respective negations and hence refute the definitions.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪; - Not Or; & And, ∧, ∩, · ; \ Not And; > Imply, greater than, →, ⇒, ↦, ≻, ⊃, ⊢, ⊨, ➞; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↚, ≲;
= Equivalent, ≡, :=, ⟺, ↔, ≜, ≈, ≃; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;
(%z<#z) C as contingency, Δ, ordinal 1; (%z>#z) N as non-contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

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The key idea is that of a first-order definable operation on a stable set lattice, an idea that goes to the heart of Kripke's semantical interpretation of the modalities □ and ◊. On the algebra of subsets of a Kripke frame (X,R), the modal connectives can be interpreted as operations assigning to each set A⊆X the sets

□A = {x :∀y(xRy→y∈A)} and (1.1)

LET p, r, s, x, y: A, R, X, x, y

#p=(((x&(r&#y))>(#y<p))>x) ; TCTC TCTC TCTC TCTC(16),
FN**F**N **F**N**F**N **F**N**F**N **F**N**F**N(16) (1.2)

◊A = {x :∃y(xRy&y∈A)}. (2.1)

%p=(((x&(r&%y))&(%y<p))>x) ; CTCT CTCT CTCT CTCT(32) (2.2)

The expressions defining the members of these sets can be seen as first order formulas in the binary predicate xRy and the unary predicate y∈A, leading to the 'standard translation' of the propositional modal language into a first-order language [...]. This ability to relate modal logic to a fragment of first-order logic does much to account for the success of the relational semantics revolution.

Remark 2.2: Eqs. 1.2 and 2.2 as rendered are *not* negations, and hence refute the definitions as a standard translation.