# Landauer's principle as a consequence of the Shannon-Hartley theorem

Abstract: Communication and information in the world of black bodies. The message, in contrast to the information, obeys the Landauer principle, according to which any generator, transmitter, and receiver of a message is a source of thermal radiation, which in the case of electromagnetic communication is conveniently considered in terms of the Planck black body model. In this case, the Shannon-Hartley theorem leads to the well-known formulation of the Landauer principle.

According to the Shannon – Hartley theorem [1], the data transmission rate over a communication channel or channel capacity is:

$$C = B \cdot \log_2\left(1 + \frac{S}{N}\right) , \qquad (1)$$

where
C is the channel capacity in bits per second, a theoretical upper bound on the net bit rate;
B is the bandwidth of the channel in hertz;
S is the average received signal power, watts (or volts squared);
N is the average power of the noise, watts (or volts squared);
S/N is the signal-to-noise ratio (SNR).

Wien's displacement law [2] allows us to represent the bandwidth in the form of:

$$B = \nu_{max} = \frac{k \cdot T}{h} \cdot \alpha \quad , \qquad (2)$$

where
$\nu_{max}$ is the frequency corresponding to the maximum of the black body radiation curve, Hz;
k is the Boltzmann constant, J / K;
T is temperature, K;
α ≈ 2.821439 ... is a constant.

Substituting (2) into (1) and multiplying both sides of the equation by the Boltzmann constant, we get:

$$W = k \cdot T \cdot \alpha \cdot \log_2(1 + SNR) \quad , \qquad (3)$$

where
W - energy of a bit in the communication channel, J.

Taking in (3) SNR = 0.1856, we obtain the well-known expression of the Landauer principle:

$$W = k \cdot T \cdot \ln(2) \quad , \qquad (4)$$

Planck's blackbody model is based on the Bose-Einstein distribution, but in some cases it is convenient to use the Fermi-Dirac distribution.

Links

1. Shannon–Hartley theorem https://en.wikipedia.org/wiki/Shannon%E2%80%93Hartley_theorem
2. Wien's displacement law https://en.wikipedia.org/wiki/Wien%27s_displacement_law