**Refutation of the power set in description logic**



**Abstract:** We evaluate four, simple axioms of any Ω-model, including the operator Pow, to support the power set in description logic. None is tautologous, meaning the power set as asserted is refuted.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪; - Not Or; & And, ∧, ∩, · ; \ Not And; > Imply, greater than, →, ⇒, ↦, ≻, ⊃, ⊢, ⊨, ➝ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲;
= Equivalent, ≡, :=, ⟺, ↔, ≜, ≈, ≃; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, ∅, Null, ⊥, zero;
(%z<#z) `C` as contingency, Δ, ordinal 1; (%z>#z) `N` as non-contingency, ∇, ordinal 2;
~( y < x) ( x ≤ y), ( x ⊆y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

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2.2 First order theory Ω

The first-order theory Ω consists of the following four (simple) axioms, written in the language whose relational symbols are ∈and ⊆, and whose functional symbols are ∪, \, Pow: (2.2.0)

> **Remark 2.2.0:** Pow is an operator derived from formal semantics of the language OWL-Full, *not* based on the corrected Square of Opposition and thus not bivalent but a probabilistic vector space.
>
> We take Pow to mean (C)∈Pow(C), where also possibly Pow(C)∈C, and map it as: (C)⊆(C), or ~((C)<(C)).

x∈y∪z↔x∈y∨x∈z; (2.2.1.1)

LET p, q, r: x, y, z

((p<q)+r)=((p<q)+(p<r)) ; `TTT`**`F`** **`F`**`TTT`**`F`** `TTT`**`F`** **`F`**`TT`**`F`** (2.2.1.2)

x∈y\z↔x∈y∧x∉z; (2.2.2.1)

((p<q)\r)=((p<q)&~(p<r)) ; **`FFFF FFFF FFFF FFFF`** (2.2.2.2)

$x\subseteq y\leftrightarrow\forall z(z\in x\rightarrow z\in y);$ (2.2.3.1)

$\sim(q<p)=((\#r<p)>(\#r<q))$ ; TT**F**T TTNT TT**F**T TTNT (2.2.3.2)

$x\in Pow(y)\leftrightarrow x\subseteq y.$ (2.2.4.1)

$(p<\sim(q<q))=\sim(q<p)$ ; **FF**TF **FF**T**F** **FF**T**F** **FF**T**F** (2.2.4.2)

**Remark 2.2:** Eqs. 2.2.n.2 as rendered are *not* tautologous. This refutes the four, simple axioms of any Ω-model.

In any Ω-model *everything* is supposed to be a set. Hence, a set will have (only) sets as its elements and circular definitions of sets are not forbidden—i.e., for example, there are models of Ω in which there are sets admitting themselves as elements. Moreover, not postulating in Ω any *link* between membership ∈and equality—in axiomatic terms, having no *extensionality* (axiom)—, there exist Ω-models in which there are different sets with equal collection of elements.