Interval Sieve Algorithm

Creating a Countable Set of Real Numbers from a Closed Interval

I. Introduction

I wrote a paper posted on viXra.org e-Print archive (**viXra:1806.0030**) titled The Function f(x) = C and the Continuum Hypothesis wherein I proposed a proof of the CH. In the paper I showed that by indexing (using a unique natural number for each index value) the calculation of the function's range values for each element of the domain I could establish a one-to-one correspondence between the domain and my index. It was pointed out that in order to present the domain of the function in the form of a list, which I needed to do in order to create my index, I first had to prove that the list contained every element in the interval from which the domain was defined.

I began working on the problem and came up with what I call the Interval Sieve Algorithm which uses a method of repeatedly dividing a closed interval into a series of closed sub-intervals and using the numbers bounding each sub-interval to form a list that will be used as the domain of $f(x) = C$.

II. Abstract

The Interval Sieve Algorithm is a method for generating a list of real numbers on any closed interval $[r_i, r_j]$ where $r_i < r_j$, which can then be defined as the domain of the function $f(x) = C$.

The purpose of this paper is to delineate the steps of the algorithm and show how it will generate a countable list from which the domain for the function $f(x) = C$ can be defined. Having constructed the list we will prove that the list is complete, that it contains all the numbers in the interval $[r_i, r_j]$.

III. Given

1. The set of natural numbers

$$\mathbb{N}, \{n \in \mathbb{N} \mid 1 \leq n\}$$

2. The set of real numbers

$$\mathbb{R}, \{r \in \mathbb{R} \mid r \text{ is real}\}$$

3. The closed interval

$$[r_1, r_2] \text{ where } r_1 < r_2 \text{ and } r_1, r_2 \text{ are real numbers}$$

4. The list

$$L = \{r_1, r_2\}$$

IV. Definitions

1. The **lower bound** of a closed interval is the smaller of the two numbers comprising the interval. In the interval $[r_1, r_2]$ where $r_1 < r_2$, r_1 is the lower bound of the interval.

2. The **upper bound** of a closed interval is the larger of the two numbers comprising the interval. In the interval $[r_1, r_2]$, where $r_1 < r_2$, r_2 is the upper bound of the interval.

4. A **conjoined interval pair** is a pair of closed intervals where the upper bound of one and the lower bound of the other are the same number. $[r_i, [r_k,] r_j]$ is an example of a conjoined interval pair where r_k is both the upper bound of $[r_i, r_k]$ and the lower bound of $[r_k, r_j]$.

5. A **relative bound** is a number that is common to both intervals in a conjoined interval pair. In the conjoined interval pair $[r_1, [r_3,] r_2]$, where $r_1 < r_3 < r_2$, r_3 is a relative bound in both intervals $[r_1, r_3]$ and $[r_3, r_2]$.

The importance of the relative bound will become apparent when we get into the description of the Interval Sieve Algorithm.

6. The **immediate predecessor** of a number λ is a number β such that there exists no number δ where $\beta < \delta < \lambda$.

7. The **immediate successor** of a number λ is a number β such that there exists no number δ where $\lambda < \delta < \beta$.

From definitions 6 and 7, for any 2 real numbers λ and β, in the interval, we can always find another real number, δ, such that if $\lambda > \beta$ then $\beta < \delta < \lambda$ and if $\lambda < \beta$ then $\lambda < \delta < \beta$.

V. The Interval Sieve Algorithm

Procedure:

0. We begin the procedure with the interval

$$[r_1, r_2] \text{ where } r_1 < r_2 \text{ and } r_1, r_2 \text{ are real numbers}$$

and the list

$$L = \{r_1, r_2\}$$

1. Sub-divide each interval $[r_i, r_j]$ by selecting a number r_k such that $r_i < r_k < r_j$ to get a conjoined interval pair:

$$[r_i, [r_k,] r_j]$$

2. Insert the relative bound number, r_k, into the list L to get

$$L = \{r_i, r_k, r_j\}$$

3. Return to step 1.

The algorithm produces the following results:

Interval Sieve Algorithm

$[r_1$																$r_2]$
$[r_1$								$[r_3]$								$r_2]$
$[r_1$				$[r_4]$				$[r_3]$				$[r_5]$				$r_2]$
$[r_1$		$[r_6]$		$[r_4]$		$[r_7]$		$[r_3]$		$[r_8]$		$[r_5]$		$[r_9]$		$r_2]$
$[r_1$	$[r_{10}]$	$[r_6]$	$[r_{11}]$	$[r_4]$	$[r_{12}]$	$[r_7]$	$[r_{13}]$	$[r_3]$	$[r_{14}]$	$[r_8]$	$[r_{15}]$	$[r_5]$	$[r_{16}]$	$[r_9]$	$[r_{17}]$	$r_2]$

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Intervals Generated by the Algorithm

$[r_1, r_2]$

[r1, r3][r3, r2]

[r1, r4][r4, r3][r3, r5][r5, r2]

[r1, r6][r6, r4][r4, r7][r7, r3][r3, r8][r8, r5][r5, r9][r9, r2]

$[r_1, r_{10}][r_{10}, r_6][r_6, r_{11}][r_{11}, r_4][r_4, r_{12}][r_{12}, r_7][r_7, r_{13}][r_{13}, r_3][r_3, r_{14}][r_{14}, r_8][r_8, r_{15}][r_{15}, r_5][r_5, r_{16}][r_{16}, r_9][r_9, r_2]$

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List of Real Numbers Generated by the Algothrim

$L = \{r_1, r_2\}$

$L = \{r_1, r_3, r_2\}$

$L = \{r_1, r_4, r_3, r_5, r_2\}$

$L = \{r_1, r_6, r_4, r_7, r_3, r_8, r_5, r_9, r_2\}$

$L = \{r_1, r_{10}, r_6, r_{11}, r_4, r_{12}, r_7, r_{13}, r_3, r_{14}, r_8, r_{15}, r_5, r_{16}, r_9, r_{17}, r_2\}$

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Beginning with one interval, growth of the number of intervals created is exponential and after the fourth iteration we have a total of 16 intervals. If n is the number of iterations and I is the number of intervals, we have $I = 2^n$ and if L_n is the number of list elements then

$L_n = 2^n + 1$.

VI. Proving the List is Complete

The question remains as to whether or not the list L will contain all real numbers in $[r_1, r_2]$. With the help of Cantor's Diagonal Argument we will prove that: **All the real numbers in $[r_1, r_2]$ are contained in the list L**.

Proof:

Let each number in L be represented by its digits so that:

$r_1 = d_1d_2d_3d_4\ldots$
$r_2 = d_1d_2d_3d_4\ldots$
$r_3 = d_1d_2d_3d_4\ldots$

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Let a vertical list of elements of L be called List B.

Using Cantor's Diagonal Argument we will generate a number X that is not contained in List B and then show that X will be contained in L.

Examining X we note that:

1. $r_1 < X < r_2$, since for any number in L, r_k, $r_1 < r_k < r_2$ and X is created from numbers in L that have been arranged vertically in List B. We know therefore that X is in $[r_1, r_2]$.

2. Since X is in $[r_1, r_2]$ then it must be either a member of a sub-interval contained in $[r_1, r_2]$ or the relative bound of 2 sub-intervals in $[r_1, r_2]$.

3. If X is a relative bound of 2 sub-intervals in $[r_1, r_2]$ it is already an element of L.

4. If X is a member of a sub-interval contained in $[r_1, r_2]$ and not a relative bound, then at some point it will become a relative bound of 2 sub-intervals contained in $[r_1, r_2]$.

5. Once X becomes a relative bound of 2 sub-intervals it will be included in L and become a member of L.

6. There are no other cases regarding the nature of X to consider, therefore at any point in time, all numbers X_i are or will be elements of L.

7. We can then assert that at infinity L will be complete and this ends the proof.