Denial of consistency for the Lobachevskii non Euclidean geometry

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Abstract: We prove two parallel lines are tautologous in Euclidean geometry. We next prove that non Euclidean geometry of Lobachevskii is *not* tautologous and hence *not* consistent. What follows is that Riemann geometry is the same, and non Euclidean geometry is a segment of Euclidean geometry, not the other way around. Therefore non Euclidean geometries are a non tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not,
$$\neg$$
; + Or, \lor , \cup ; - Not Or; & And, \land , \cap , \cdot ; \ Not And;
> Imply, greater than, \rightarrow , \Rightarrow , \mapsto , \succ , \supset , \neg ;
< Not Imply, less than, \in , \prec , \subset , \nvdash , \nexists , \leftarrow , \lesssim ;
= Equivalent, \equiv , :=, \leftrightarrow , \leftrightarrow , \triangleq , \approx , \simeq ; @ Not Equivalent, \neq , \sqcup ;
% possibility, for one or some, \exists , \diamond , M; # necessity, for every or all, \forall , \Box , L;
(z=z) T as tautology, \top , ordinal 3; (z@z) F as contradiction, Ø, Null, \bot , zero;
(%z>#z) N as non-contingency, Δ , ordinal 1;
(%z<#z) C as contingency, ∇ , ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: encyclopediaofmath.org/index.php/Lobachevskii_geometry

A [Lobachevskii] geometry [is] based on the same fundamental premises as Euclidean geometry, except for the axiom of parallelism.

Lobachevskii geometry is the geometry of a Riemannian space of constant curvature. The proof of the consistency of Lobachevskii geometry is carried out by constructing an interpretation (a model).

Consider the Euclidean polygon below with (p,q), (r,s) parallel to (t,u), (v,w):

$$\begin{array}{c|c} (p,q) & (r,s) \\ | & | \\ | & | \\ (t,u) & (v,w) \end{array}$$

If u is less than q and w less than s, and q is equivalent to s and u equivalent to w, and p is less than r and t less than v, then q minus u is equivalent to s minus w (thereby keeping the edge (p,q), (r,s) parallel to (t,u), (v,w)). (1.1)

$$((((u < q) \& (w < s)) \& ((q = s) \& (u = w))) \& ((p < r) @ (t < v))) > ((q - u) = (s - w));$$

TTTT TTTT TTTT TTTT ((1.2)

Remark 1.2: Eq. 1.2 is tautologous and hence consistent.

The non Euclidean spherical geometry of Lobachevskii asserts parallel lines intersect at some point and hence are not parallel at that point. (2.0) **Remark 2.0:** We note that Eq. 2.0 also applies to hyperbolic non Euclidean geometry. If u is less than q and w less than s, and q is equivalent to s and u equivalent to w, and p is less than r and t less than v, then q minus u is *not* equivalent to s minus w (thereby *not* keeping the edge (p,q), (r,s) parallel to (t,u), (v,w)). (2.1) ((((u < q)&(w < s))&((q = s)&(u = w)))&((p < r)@(t < v))) > ((q - u)@(s - w));TTTT TTTT TTTT TTTT (10), TFTT TTTT TTTT TTTT (1), TTTT TTTT TTTT (2), TFTT TTTT TTTT (2), (2.2)

Remark 2.2: Eq. 2.2 as rendered is *not* tautologous, meaning Eq. 2.0 is *not* consistent.

What follows is that Riemann geometry is the same, and non Euclidean geometry is a segment of Euclidean geometry, not the other way around. Therefore non Euclidean geometries are a non tautologous fragment of the universal logic VŁ4.